

# DESIGN OF COMPOSITE HIGHWAY BRIDGES CURVED IN PLAN





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**D C Iles** MSc CEng MICE





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# FOREWORD

This publication has been written to supplement the SCI design guides *Composite highway bridge design* (P356) and *Composite highway bridge design: Worked Examples* (P357) for the design of bridges in the UK in accordance with the Eurocodes. It deals with the additional effects that need to be considered in design when a bridge is curved in plan, which is a fairly common design situation.

Some of the design rules in the Eurocodes do not explicitly cover the additional effects that arise due to curvature, although the principles are established. The objective for this publication was therefore to offer guidance, interpretations and suggested procedures for such situations.

During the preparation of the guide, advice was obtained from the members of the Steel Bridge Group and in particular from Chris Hendy, chairman of SBG. Assistance with modelling the curved bridge was provided by Bestech. These contributions are gratefully acknowledged.

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# SUMMARY

Many highway bridges carry roads that are on a curved alignment and the supporting structure follows that curved alignment. This design guide addresses the consequences of the plan curvature on the design of composite bridges and effectively offers a supplement to SCI design guides P356 and P357, where comprehensive guidance is expressed in relation to bridges that are straight in plan, for design in accordance with the Eurocodes. This new guide describes the two options of using a series of straight girder lengths (chords to a curve) and of using curved girders; it explains that curved girders is generally the favoured option and the fabrication of such girders is readily achievable in modern workshops.

The behaviour of curved elements is discussed, noting the torsional effects that arise, and the application of the Eurocodes to situations that are not always explicitly covered by its rules is considered. Consequences for construction are mentioned and the options for bridge articulation are presented.

An Appendix illustrates the application of design rules to a two-span bridge, similar to that in SCI publication P357 but curved in plan. The Appendix compares key differences between the effects for straight and curved configurations.



# INTRODUCTION

## 1.1 The need for curvature in plan

Inevitably, many highway bridges carry roads that are on a curved alignment. Lower speed roads and link roads may have a radius of curvature of as low as 90 m, which leads to a significant angular change in the direction of the carriageway, even over short spans, and this will have a significant effect on the design and construction of the bridge. Higher speed roads have a greater radius but then may require longer spans and the curvature will still have a significant effect on design and construction.

In the past, composite bridge design usually responded to the need for curved road alignment by using a series of straight girder sections, forming a sequence of chords to a curve. Fabrication of girders curved in plan was more difficult and costly; analysis of a curved configuration was considered to be complex.

With modern fabrication equipment, curvature in plan does not pose any major difficulties; with the analytical software now available, analysis of curved configurations is relatively straightforward. Contractors prefer curved girders because it simplifies fixing reinforcement and formwork. And since “true” curvature generally has a much more natural appearance than a series of straights, clients’ preferences are for truly curved girders for such bridges.

## 1.2 Scope of this publication

Design guidance for composite highway bridges is given in SCI publication P356 and that guidance is complemented by the worked examples in Publication P357. Those two books cover the design of slab on beam composite bridges of multi girder and ladder deck configurations, using I section main girders. All the guidance in those publications, including advice on the application of the Eurocodes for design verification, remains valid where the girders are curved in plan but, in addition, the effects of curvature need to be taken into account. The present publication provides the additional guidance for such configurations. The design of box girders is outside the scope of P356 and is also outside the scope of the present publication.

### 1.3 Straight or curved girders?

For single-span bridges with a curved deck, the alternative options would normally be either straight girders over the whole span or a curved girder. A key consideration would therefore be the offset (of the curve from the chord) at mid-span – typical values are shown in Table 1.1. Large values of offset would lead to significant variation in cantilever length if straight girders were used, resulting in complexity in formwork and fixing reinforcement; main contractors therefore prefer truly curved girders.

Table 1.1  
Typical offsets  
(curve to chord) for  
single spans

SPAN LENGTH (m)	OFFSET (m) FOR RADIUS OF CURVATURE (m)			
	180	300	500	800
20	0.278	0.167	0.100	0.063
30	0.625	0.375	0.225	0.141
40	1.111	0.667	0.400	0.250

For multiple spans, it is common to arrange girder splices at the points of contraflexure (between the  $\frac{1}{3}$  and  $\frac{1}{4}$  points in the span). If straight girders are chosen, then it would be convenient to make the angular changes at these positions (it is uneconomic to make angular changes other than at splice locations). For two angular changes in each span, the change at each position for typical spans is given in Table 1.2.

Table 1.2  
Typical angular  
change at a splice  
between straight  
girders, when there  
are two splices in  
each span

SPAN LENGTH (m)	ANGULAR CHANGE (degrees) FOR RADIUS OF CURVATURE (m)			
	180	300	500	800
20	3.2	1.9	1.1	0.7
30	4.8	2.9	1.7	1.1
40	6.4	3.8	2.3	1.4
50	8.0	4.8	2.9	1.8
60		5.7	3.4	2.1

Generally, truly curved girders give a much better appearance for multiple spans, as illustrated in Figure 1.1, and they allow constant length cantilevers. Nevertheless, the use of a series of straight girders may give a perfectly acceptable appearance in many situations, especially where the radius is large and the angular changes at splices are modest.

### 1.4 Highway geometry

Depending on the design speed of the road and the length of the curved alignment, super-elevation may be required for the carriageway. This will have consequences on the detailing of the deck slab and the girders – see discussion in Section 6.4. It is unlikely that a plan radius of less than 180 m would be required on a bridge (and that radius would only be needed for low road design speeds and would require super-elevation).

If there is a transition curve in the highway alignment on the bridge, the curvature will vary over the length of the bridge. Girders can be shaped to follow the varying curvature, although the difference from a uniform curve will usually be small.

## 1.5 Typical configurations

Typical examples of bridges where the girders are curved in plan are shown in the Figures 1.1 to 1.5.



Figure 1.1  
Hunslet Viaduct  
(ladder deck bridge)

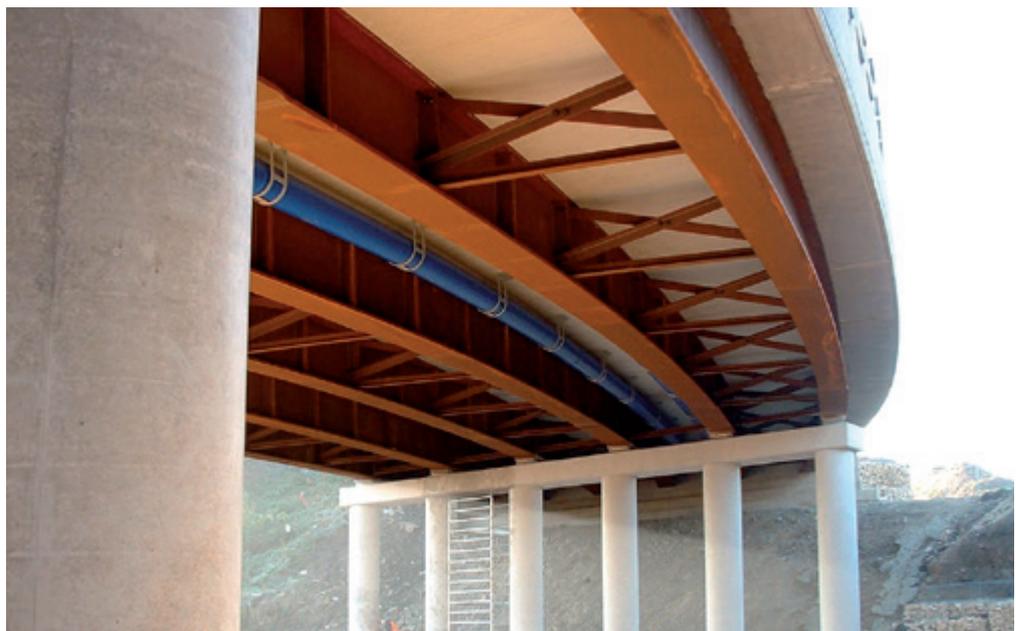


Figure 1.2  
Highfield Lane Bridge



Figure 1.3  
M25/A2  
interchange bridge



Figure 1.4  
Erection of curved  
girders, Bargoed  
Regeneration Scheme



*Figure 1.5  
M50 Overbridge,  
Dublin  
(plan radius 185 m)*



# THEORETICAL BACKGROUND

Vertical loads on curved beams inevitably lead to torsional moments as well as bending and shear. It is therefore helpful to look first at how torsion is resisted in straight beams.

## 2.1 Behaviour of straight beams

The behaviour of straight beams subject to vertical loading is well established and appreciated by designers. Generally, 'engineer's theory of bending' assumes that plane sections remain plane and that the loading is resisted by (longitudinal) direct stresses, expressed as the bending moment at a cross section. Variation of longitudinal stress / bending moment along the beam is associated with shear stresses that are uniform across the thickness of the element of section at any point in the cross section.

A uniform beam that is subject to constant torsion will twist about a longitudinal axis and the ends of the beam will warp out of plane, as shown for an I section in Figure 2.1.

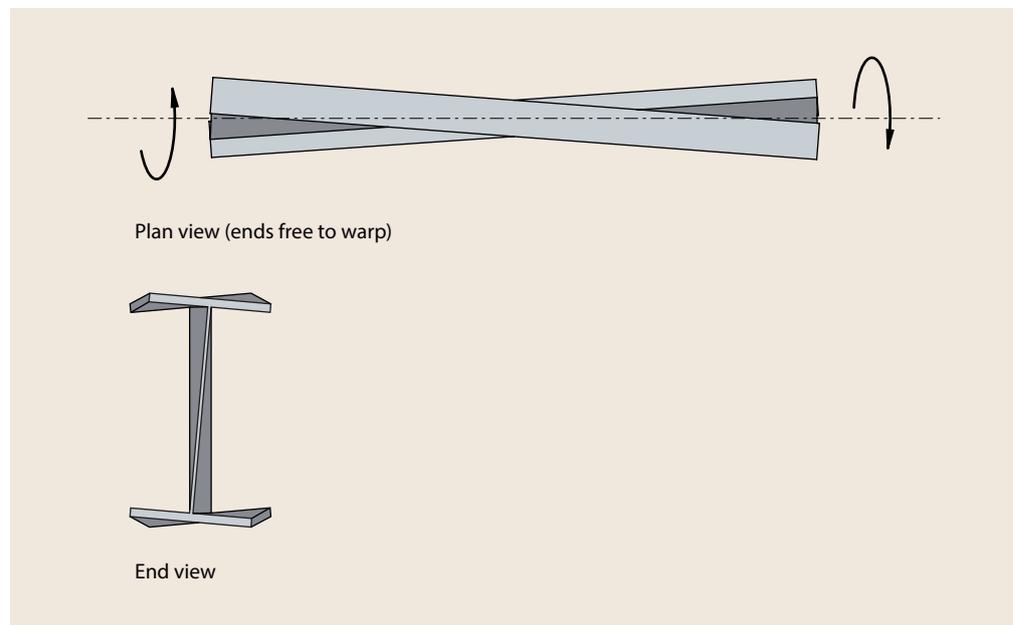


Figure 2.1  
Beam subject to  
uniform torsion

Any straight line on the surface of the beam will remain straight; there are thus no longitudinal (direct) stresses and the torsion is resisted by a pattern of shear stresses known as St Venant torsional shear stresses. In an open section (an I beam for example) these stresses vary through the thickness at any point in the cross section, with peak stresses in opposite directions on each face.

When the beam is subject to non-uniform torsion, the warping that would occur with St Venant torsion is constrained and longitudinal warping stresses develop. These warping stresses correspond to in-plane bending of the elements and can be readily appreciated by considering the case of a torque applied at mid-span of a simply supported beam. At mid-span, warping is fully constrained (by consideration of symmetry) and the flanges bend over the length of the beam, as shown in Figure 2.2. As a simplification, if it is assumed that St Venant torsional stiffness is negligible, the two flanges would bend in their planes as a simply supported beam, each subject to a force that is equal to the torque divided by the distance between the flange centroids. At mid-span, there is a warping moment in each flange and the torsion is fully resisted by the associated shear stresses.

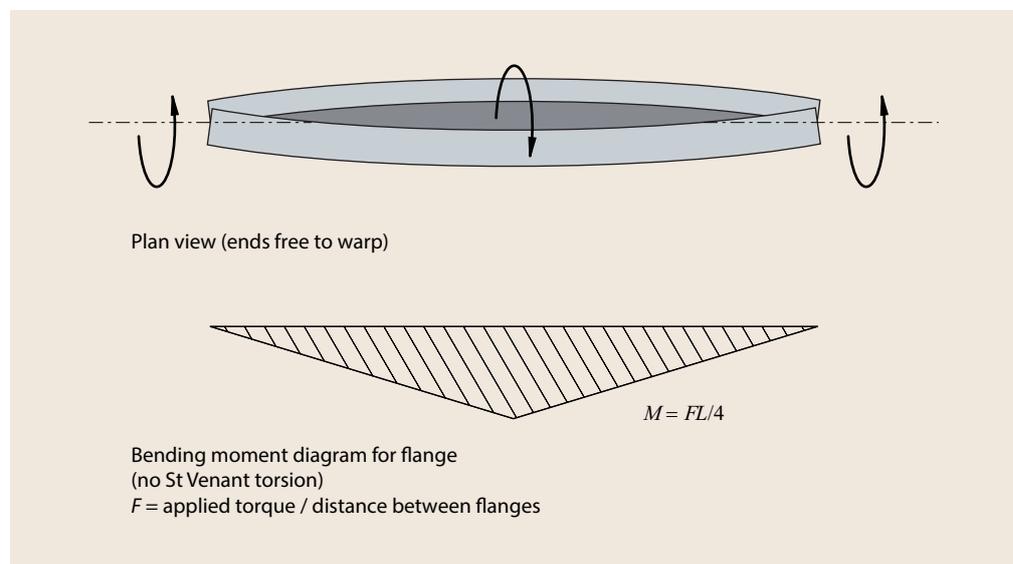


Figure 2.2  
Beam subject  
to a centrally  
applied torque

In practice, the St Venant torsional stiffness is not negligible and for long beams, the warping shear stresses diminish towards the ends and the torsional moment near the ends is almost entirely St Venant torsional moment. However, for short beams warping shear stresses will still be significant at the ends and the St Venant torsional moment will be small.

The distinction between 'long' and 'short' beams is indicated by the torsional bending constant  $a (= \sqrt{EI_W / GI_T})$ . For a discussion of this parameter, see P385<sup>[4]</sup> but for the present purposes its significance might be judged by considering a point torque applied at the centre of a simple beam where warping is not restrained at the ends. If the beam length is equal to  $6a$ , the warping torsional moment at the ends (i.e. at  $3a$  from the application of the point torque) is less than 10% of the total torsional moment – see Figure 2.3.

Similar examples can be derived for uniformly distributed torque and for warping restraint at the ends, all of which demonstrate that torsion is resisted as warping torsion when the length is a small multiple of  $a$  and as St Venant torsion when the length is a large multiple of  $a$ .

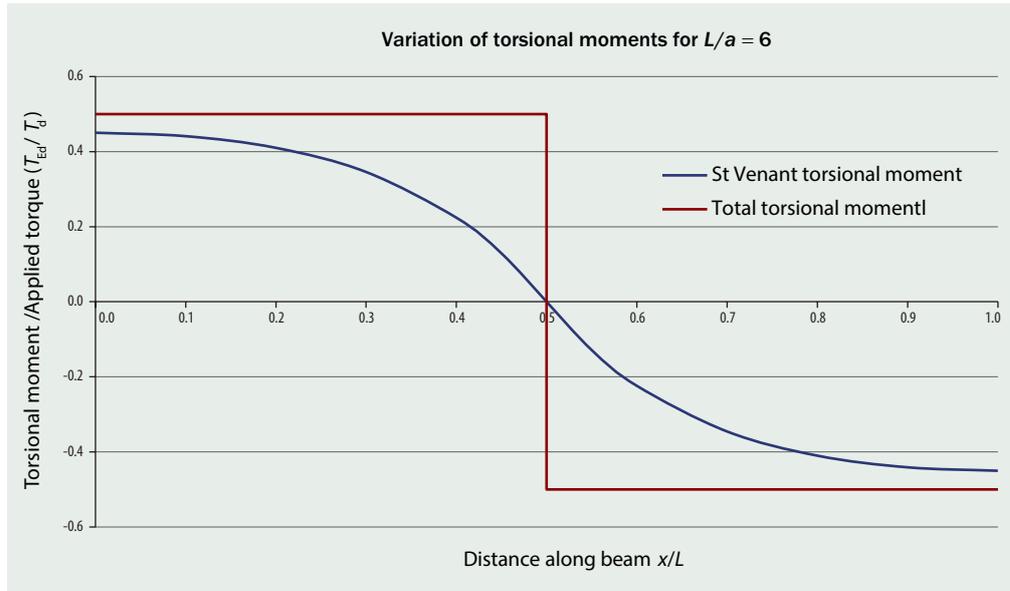


Figure 2.3  
Variation of torsional moments along a beam subject to a central applied torque

Typically, bridge girders have rather heavy flanges and the torsional constant is quite large, of the order of several metres. Values for several example girder sections are given in Table 2.1.

Table 2.1  
Values of torsional bending constant ( $a$ ) for typical girder sections

	EXAMPLE 1	EXAMPLE 2	EXAMPLE 3
Girder depth (mm)	1100	1450	2000
Top flange (mm × mm)	500 × 40	500 × 30	650 × 70
Web (mm)	12	25	22
Bottom flange (mm × mm)	500 × 40	600 × 45	700 × 65
Parameter $a$ (mm)	2635	3230	3835

## 2.2 Behaviour of a curved beam

### 2.2.1 Single girders

In a beam that is curved in plan and subject to vertical loading, longitudinal bending stresses develop to the same extent as in a straight beam but, since the plane of the bending moment is continuously changing along the curve, it is associated with an effective torque (per unit length) given by  $T = M_y/R$  (where  $M_y$  is the bending moment about the y-y axis and  $R$  is the radius of curvature of the beam). For an I section beam this can perhaps be most easily appreciated as equivalent to a lateral (or radial) force per unit length  $F = T/h_f = M_y/Rh_f$  where  $h_f$  is the distance between the flange centroids.

As for straight beams, the balance between St Venant torsional moment and warping torsional moment depends on the torsional bending constant  $a$ .

As a guide, Figure 2.4 shows the variation of torsional moment in a simply supported beam of length  $6a$  and subject to constant torque per unit length over the whole span. It can be seen that St Venant torsion provides the major component of the torsional moment at the ends. The corresponding warping moment in each flange is greatest at

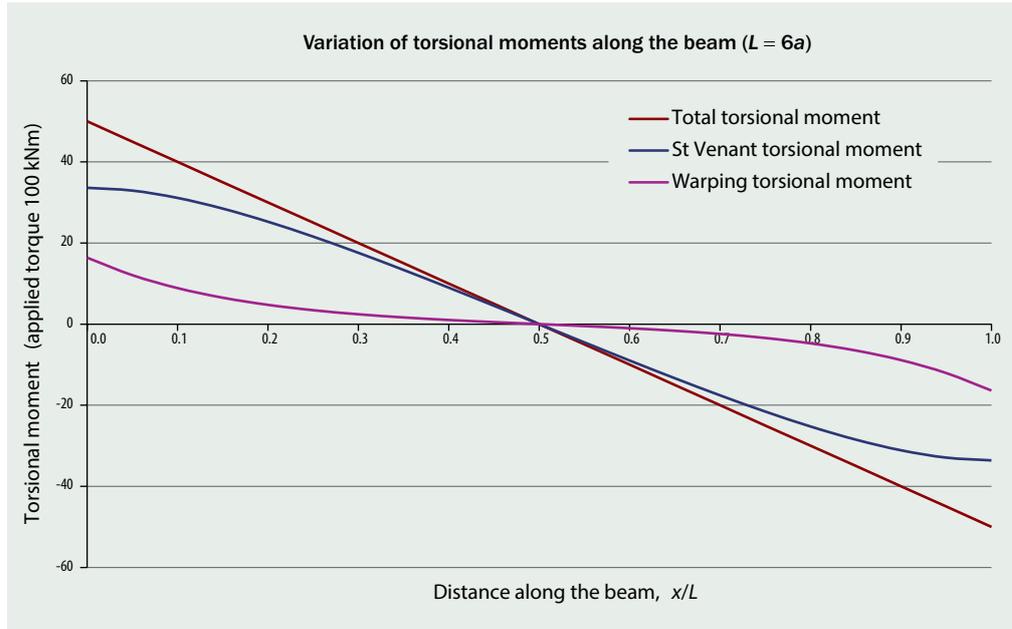


Figure 2.4  
Variation of torsional moments along a beam subject to uniformly distributed torque (simple span, free to warp at ends)

mid-span (see Figure 2.5) but it is only about 20% of that of a simplified model, neglecting St Venant torsion (which would then have a value of  $FL/8$ , where  $F = T/h$ , in which  $T$  is the total torque and  $h$  is the distance between the flange centroids).

For a curved beam under vertical loading, the torque per unit length varies in proportion to the vertical bending moment. In consequence, the St Venant torsion at the ends would be a slightly greater proportion of the total torsional moment and the warping moment in the flange would be a slightly lower proportion of that for the case of constant torque per unit length. However, it is difficult to calculate the values of the two components algebraically for non-uniform torque and for actual situations a 3D computer model would be needed.

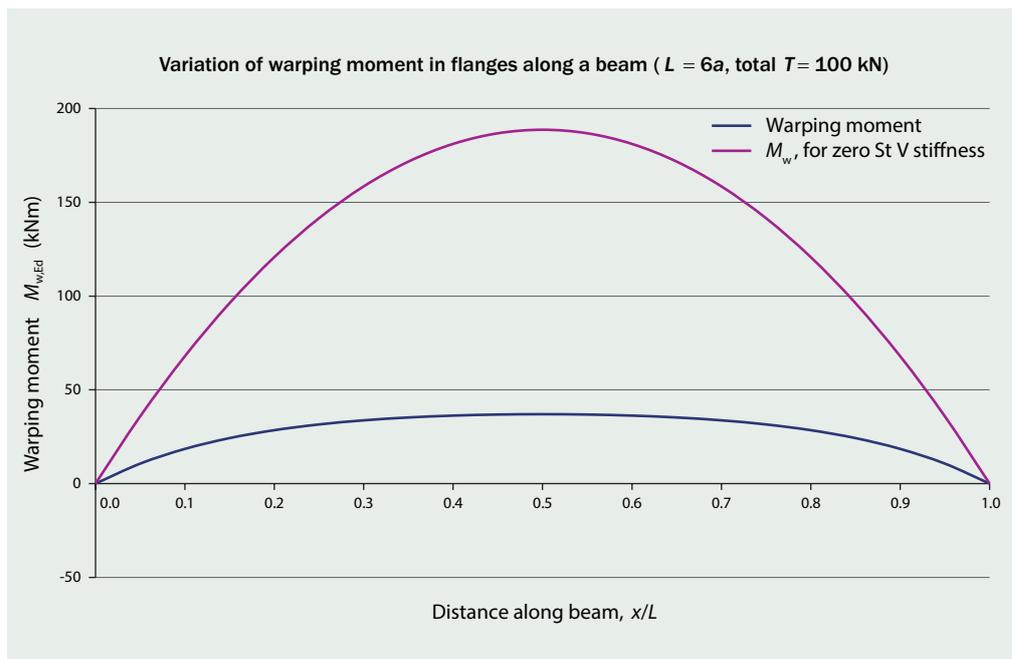


Figure 2.5  
Variation of warping moment in flanges along a beam subject to uniformly distributed torque (simple span, free to warp at ends)

The above discussion relates to elastic behaviour of the beam. It is inappropriate to consider plastic behaviour, since the plasticity would increase the rotation significantly, resulting in non-negligible minor axis bending (because the beam axis is then significantly non-vertical).

### 2.2.2 Braced girders

For a practical size of a single girder and span, where the span is likely to be in excess of  $6a$  (see Table 2.1 – for a span/girder depth ratio of 25, the spans would significantly exceed  $6a$ ), the twist of a single curved beam is likely to be excessive and it would be normal practice to pair beams by bracing them together at intervals, as shown in Figure 2.6. This achieves fully effective (or at least near-fully effective) torsional restraint to the individual girders at discrete positions: over the whole span it creates a compound section for torsion.

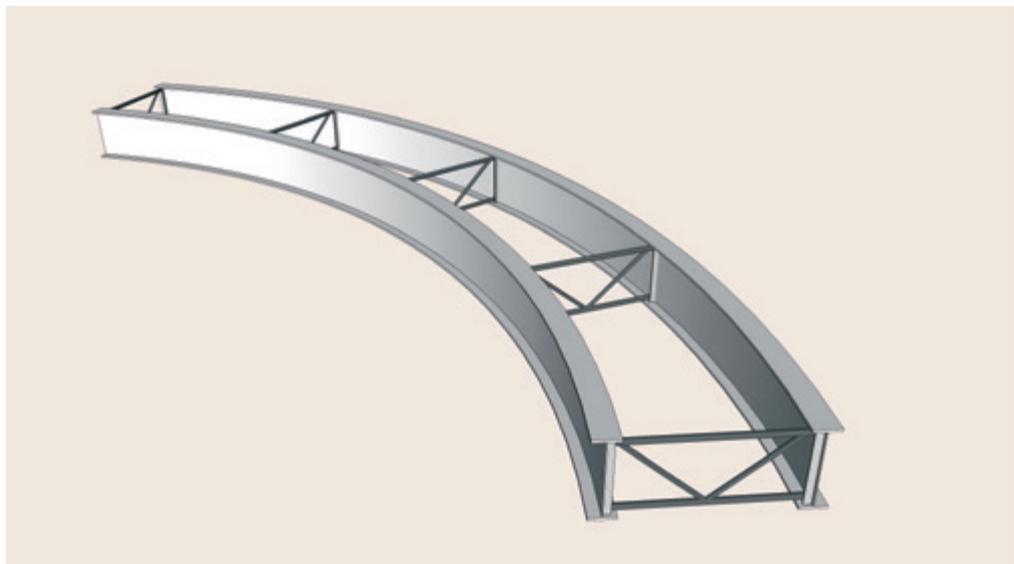


Figure 2.6  
Braced pair of girders

For the compound section, the warping stiffness is greatly increased because it depends on the vertical bending stiffness of each beam and the distance between them. The torsional bending constant  $a$  becomes large, usually greater than the span, and thus global torsional effects are resisted mainly by vertical bending of the beams. St Venant torsion is negligible.

Between the restraints, which have a spacing equal to only a small multiple of dimension  $a$ , the torsional effects will be almost exclusively plan bending of the flanges, as for a short single I beam subject to torsion; the warping moment may simply be determined by considering the radial force in each flange as a force on a continuous beam (the flange bending in its plane) spanning between the restraint positions.

The total effects in the beam flanges may therefore be evaluated with reasonable accuracy as the sum of two components – one due to vertical bending over the span and one due to lateral bending between restraint positions. For the former, the torsional actions are those due to their eccentricity from a straight line between

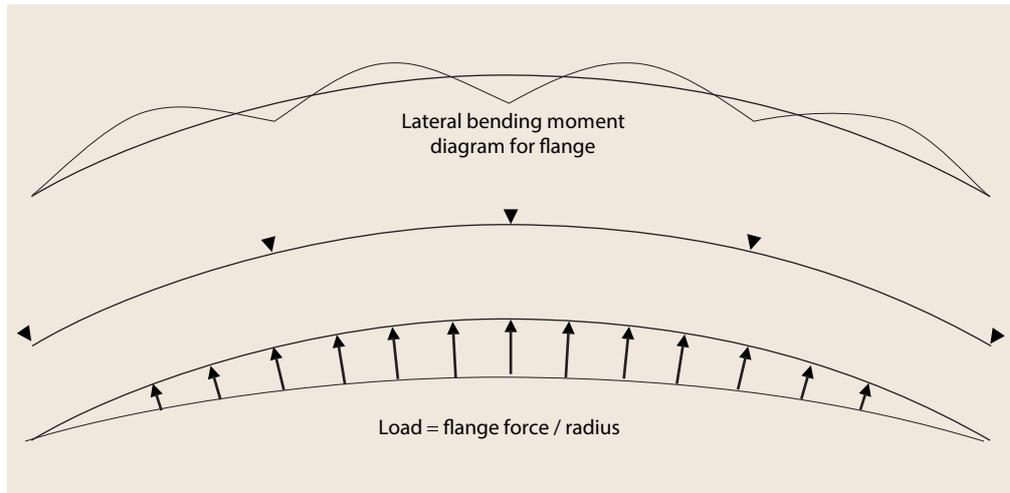


Figure 2.7  
Lateral effects in a  
flange of beam in  
Figure 2.6

supports and the effects are additional vertical bending moments (increasing the design moment in the girder on the outer side of the curve, reducing the design moment on the inner side). For the latter, St Venant stiffness may be neglected and the design moments determined for a continuous beam, as shown in Figure 2.7.

Use of an FE model will give the vertical bending effects, as stresses in the elements, directly, without the need for manual evaluation and the use of such a model is recommended. Lateral bending effects will be given by such a model if the mesh is sufficiently fine to model the curvature between restraint positions, or may be determined separately from a line beam analysis of the flange.

### 2.3 Behaviour of a straight composite deck

The behaviour of a composite bridge deck (slab on top of straight girders), of either multi-girder form or ladder deck form, is essentially that of a number of adjacent composite beams. Transverse bracing and the flexural stiffness of the slab will result in some load transfer between the beams (when the load on each is not identical) but there is no plan bending of the slab itself and very little plan bending of the bottom flanges (except in ladder deck bridges when one cross girder is much more heavily loaded than its neighbours). The longitudinal stresses are thus due to the vertical bending moment resisted by the composite cross section of one beam and an effective width of the slab.

### 2.4 Behaviour of a curved composite deck

Considering the deck as a whole, the effects on it are similar to those on a single curved beam, i.e. vertical bending moments plus torsional moments due to the curvature. However, like the compound paired girder section described above, the composite deck has a much greater warping stiffness than a single beam because any twist would require vertical bending of the beams and the horizontal bending of the slab (somewhat similar to the effect in a channel with its web horizontal).

Like the compound paired beams, although the resistance to twist is due mainly to warping stiffness, it will be more readily seen as a transfer of vertical loading from one beam to another. However, as well as the overall twist, there are distortional effects because the 'radial' forces in the bottom flange try to change the geometry of the cross section. The stiffness and strength of the web, bending out of its plane (even when there are transverse web stiffeners) is generally too weak to restrain the effects due to typical plan curvature but the paired bracing that is needed for the bare steel condition is readily able to provide the restraint. (This applies even for ladder deck bridges, although the cross girders are more flexible than triangulated bracing). As for the paired girders, the plan bending of the bottom flange of the composite section depends on the spacing of the bracing or cross girders; the restraint forces in the bracing also depend on this spacing. The lateral bending effects may be calculated manually, treating the bottom flange as a continuous beam and taking the radial force as the flange force divided by the radius. Alternatively, the lateral bending effects can be determined in an FE model, provided that the curvature between restraints is modelled with a sufficiently fine mesh.



# ANALYSIS MODELS

## 3.1 Line beams

For single unbraced curved beams at the erection stage, vertical bending moments and shears are easily calculated but torsional effects are less easy to determine. As explained in Section 2.2, the applied torque per unit length at any position (or radial force per unit length in the flange) depends on the vertical bending moment at that position and thus it varies along the whole length of the beam.

The interaction between St Venant torsion and warping torsion can be evaluated theoretically but this requires evaluation of hyperbolic functions and is thus somewhat complex. SCI publication P385 explains the behaviour and offers equations and graphs but since the applied torque is non-uniform, exact values of twist and warping moment in the flange are difficult to determine. The simplification of ignoring St Venant torsion for open sections will give very conservative (much larger) values of both twist and warping moments in the flanges (because the span is typically much greater than the torsional bending constant).

If an accurate estimate of twist at the bare girder stage is required, it may be best to create a 3D shell model that will properly account for St Venant and warping stiffnesses.

## 3.2 2D grillages – multi girder bridges

2D grillage models have traditionally been used to analyse straight multi girder composite bridges. They give realistic values of bending moments and shears on the effective section of each of the beams (girder plus slab). Differential loading (more load on one side of the deck than on the other) results in different moments in each beam and some modest torsional effects in the slab. The modest twists associated with differential deflections of adjacent girders do induce plan bending on the bottom flanges but these effects are not given by the analysis; they are however sufficiently small that they can usually be neglected.

For multi-girder bridges curved in plan, 2D grillages can be used successfully, in many circumstances. Recognising that 2D beam elements are unable to model warping torsion, a change of direction of consecutive beam elements representing the main beams should only occur at bracing positions. Then the ‘transverse’ moment associated with the change of direction of the main beam elements is resisted by beam elements

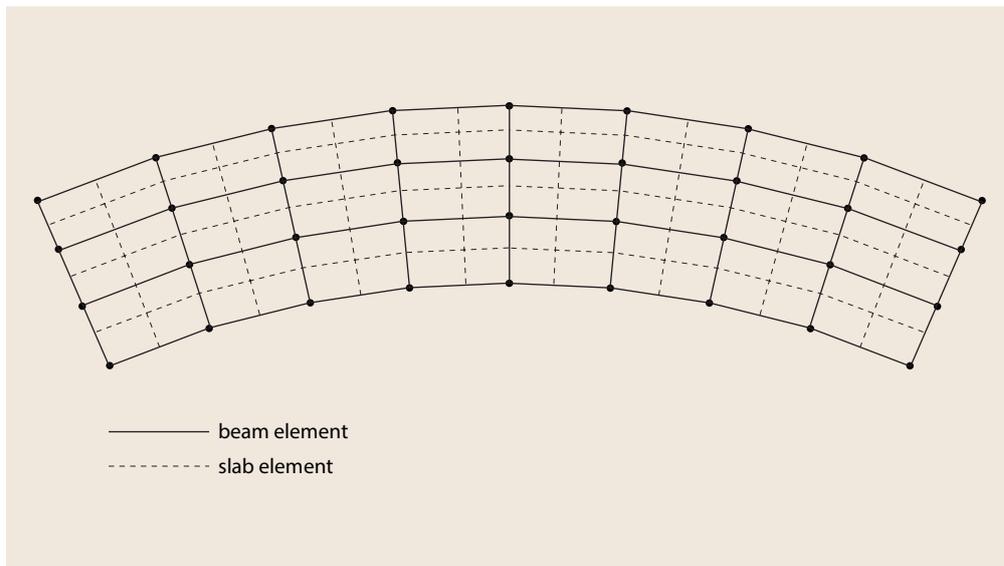


Figure 3.1  
Grillage for a  
curved multi-girder  
bridge deck

representing the bracing. An example of such a grillage is shown in Figure 3.1. In this example the slab mesh has been subdivided between bracing but if the bracing were closer (perhaps to suit a lesser radius) it might not need to be subdivided.

Such a mesh is suitable for both the bare steel configuration (even if the bracing is not continuous between pairs) and the composite configuration. The aspect of behaviour that is not modelled is the plane bending in the flanges between bracing positions; this can be evaluated manually and has no effect on the overall deflection and twist. The plan bending of the flanges over the whole span as the result of twist has also not been modelled; in most practical cases the moment in the flange due to that effect would be very small and the stiffness against such bending would reduce twist only very slightly.

If the curvature is small, it would be possible to use a ‘straight’ grillage, ignoring the curvature of the beams. Such a model can be used if the subtended angle over the span is within the limits in Table 3.1, according to recommendations in the AASHTO bridge specification<sup>[2]</sup>.

Table 3.1  
Recommended limits  
to angle subtended  
in plan for modelling  
with a straight grillage  
(from AASHTO)

NUMBER OF GIRDERS	ANGLE FOR 1 SPAN	ANGLE FOR 2 OR MORE SPANS
2	2°	3°
3 or 4	3°	4°
5 or more	4°	5°

### 3.3 2D grillages – ladder deck bridges

As explained in P356, for straight bridges, 2D grillage models are less well suited to ladder deck bridges because the deflections of the U frames formed by the cross girders and web stiffeners result in significant distortion of the cross section and the variation of that distortion from one frame to the next causes lateral bending of the bottom flange (which is a form of warping). These effects (mainly the effects due to one

cross-beam being more heavily loaded than its neighbours) would have to be evaluated manually, or by a local skeleton model, as discussed in Section 5.2.2 of P356, and added to the global results. The distortional effect is illustrated in Hendy and Murphy<sup>[3]</sup>.

Perhaps surprisingly, torsional effects due to curvature would be adequately modelled by a 2D grillage model, provided that the main girder is modelled as chords to the curve, between cross frames. The wide spacing of the main girders, coupled with the stiffness of the U frames, will resist the torsion as differential bending/warping torsion and there will be very little St Venant torsion, even in the slab. However, the complexity of needing to evaluate separately the lateral bending of the flanges between cross frames due to their different distortions remains and to those effects must be added the local bending due to the effect of curvature. 2D grillages are rarely suitable for curved ladder decks.

### **3.4 3D space frame models**

3D models constructed solely from beam elements are more able to demonstrate the 3-dimensional deflections of multi-girder and ladder deck bridges but care is needed in choosing representative elements and assigning appropriate stiffnesses to each of the elements. See comment in Section 5.2.2 of P356. The representation of the behaviour of transversely stiffened webs is a particular difficulty. The output forces and moments in the beam elements of the model need to be converted into equivalent forces and moments in the steel and composite sections for verification against design rules. Such models should only be used by designers with previous experience.

A 3D shell model will in most cases be easier to construct and will give more reliable results.

### **3.5 3D shell element models**

3D models using shell elements, possibly in combination with beam elements, are much more capable of demonstrating the 3-dimensional deflections in multi-girder and ladder deck bridges, for straight bridges as well as for curved bridges.

Generally, shell elements should be used for the deck slab and for beam webs but the flanges of the beams can be adequately modelled as beam elements. Transverse web stiffeners and triangulated bracing can also be modelled using beam elements.

Ideally, the mesh needs to be sufficiently fine that lateral bending of curved flanges between restraints can be modelled. Alternatively, the flanges may be straight between restraints (with either no intermediate nodes or just one or two nodes); this will be adequate for vertical bending effects but the lateral bending between restraints, illustrated in Figure 2.7, will then need to be evaluated manually and added to the global effects.

The lateral forces on the restraints is given with sufficient accuracy either by a fine mesh model of the curved flange or by a straight element between restraints.

(If a lateral bending model is used for calculating bending in the flange, the support

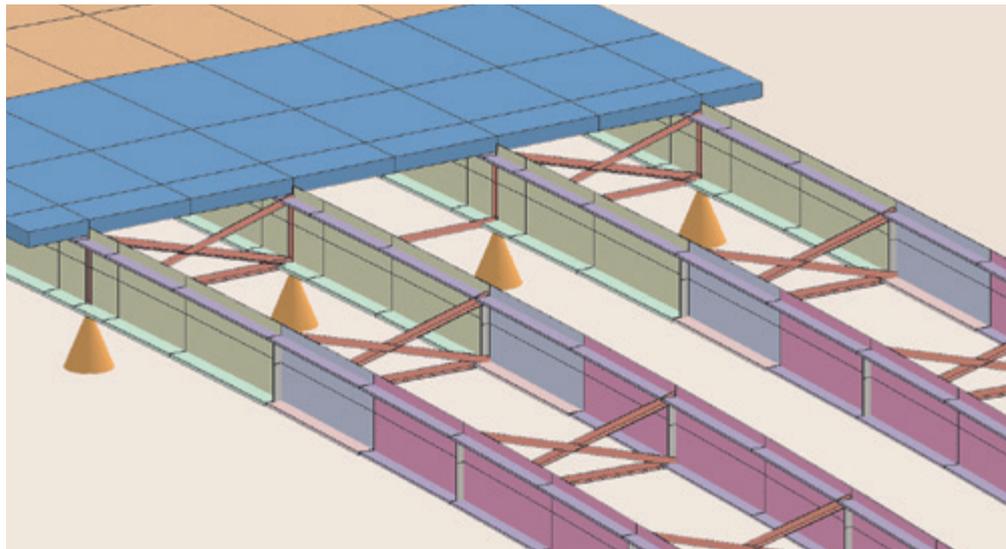


Figure 3.2  
3D model using shell  
and beam elements

Notes: Webs are modelled as shell elements, flanges and bracing as beam elements.  
For ease of modelling, bracing elements intersect at nodes and the web elements connect to the deck slab finite elements at mid-thickness of the slab.  
Colours relate to element properties.

reactions do not need to be added to the forces in the global model.) In most cases, the effects in the restraints will be small, often negligible.

Use of shell elements, with or without beam elements, will require post-processing of the structural analysis, to present the effects in terms of moments and forces that can be verified against the design rules. St Venant torsional effects in the steel beam cannot normally be extracted by such software but they are very small and can be ignored in the verification. Torsional effects in the slab may need to be considered as part of the slab design but, again, the effects are usually small.





# STRENGTH VERIFICATION (TO EUROCODES)

The verifications described in this Section relate to design in accordance with the Eurocodes. The 'lead' Eurocode Part for steel bridges is EN 1993-2, published in the UK as BS EN 1993-2, together with its UK National Annex. Many of the clauses in EN 1993-2 simply implement rules in the general Parts of Eurocode 3. For simplicity, reference is made to the Parts where the rules are given in full and the designation is made here in the form "see 3-1-1/6.3.1", meaning see clause 6.3.1 in BS EN 1993-1-1.

More detailed general guidance is given in P356.

## 4.1 Resistance of cross sections

### 4.1.1 Bending resistance

The bending resistance of a cross section depends on its classification, as given by 3-1-1/5.5.2, which classifies each element of cross section that is in compression. The classification system presumes an initially flat element, within the tolerances of the appropriate standard. The in-plane curvature of the flanges does not affect their classification. For bridge beams, the out-of-plane curvature of the web is within the flatness tolerance for plate elements<sup>1</sup> and is all in one direction; it is considered that the classification is not affected by the curvature.

The bending resistance of the cross section is given by 3-1-1/6.2.5.

### 4.1.2 Shear resistance

Generally, the webs of bridge beams will have a slenderness  $h/t_w$  greater than  $72\eta\varepsilon$  and thus, according to 3-1-1/6.2.6(6), the shear resistance must be determined in accordance with EN 1993-1-5. The design resistance for shear given by 3-1-5/5.2 takes account of shear buckling resistance and post-buckling resistance. The rules presume that the web plate is flat.

Numerical studies have shown that the elastic critical shear resistance for a curved web panel is greater than that for an equivalent straight panel. In effect, this means that a curved panel will have a lower slenderness than an equivalent straight panel.

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<sup>1</sup> For example, if a 1500 mm deep girder has a stiffener spacing of twice its depth (likely to be an upper bound in hogging regions, for economy), a plan curvature of 150 m radius would result in an out-of-straight of 7.5 mm between stiffeners, when the tolerance in BS EN 1090-2 for such a case (Table D.1.4) is  $a/250 = 12$  mm.

However, the reduction factor for a given slenderness is found to be more severe for a given curved panel than for its equivalent straight panel. This is because curved panels have less post-buckling reserve. Typically this means that the ultimate resistance of equivalent straight and curved panels are, in reality, very similar for the amounts of plan curvature typically found in bridge beams. There is however insufficient test evidence to confirm that this is always so.

Consequently, it is recommended that the ultimate shear resistance of a curved web panel is conservatively taken as the lesser of the elastic critical buckling shear force (divided by  $\gamma_{M1}$ ) for an equivalent straight web panel and the buckling resistance (given by 3-1-5/5.2 expression (5.2)) for a flat panel. The elastic critical shear force may be taken as  $h_w t \tau_{cr}$  (where  $\tau_{cr} = k_\tau \sigma_E$  and both  $k_\tau$  and  $\sigma_E$  are given by Annex A of EN 1993-1-5). For slenderness below about 1.2 the resistance is less than the elastic value and below 0.83 the resistance is limited to the plastic shear resistance. This approach is essentially that given in AASHTO<sup>[2]</sup>. It should be recognised that this approach is likely to be very conservative for slender panels.

#### 4.1.3 Torsional resistance

Where torsional effects are also present, 3-1-1/6.2.7(1) simply requires that  $T_{Ed}/T_{Rd} \leq 1$  (where  $T_{Ed}$  is the design torsional moment and  $T_{Rd}$  is the design torsional resistance) but does not give a rule for evaluating  $T_{Rd}$ . 3-1-1/6.2.7(1) says that  $T_{Ed}$  has two components, St Venant torsional moment and warping torsional moment (as discussed in Section 2.1). However, as noted earlier, for composite beams formed from I section girders, the St Venant torsional moments are in many cases small and thus the simplification permitted by 3-1-1/6.2.7(7) that the St Venant effects may be neglected for open sections is appropriate. In practice, it is difficult to extract even the warping torsional moment for these composite beams from the global analysis, since the results simply give total moment and total shear on each beam.

#### 4.1.4 Combined bending, shear and torsion

Combined bending and shear is covered in 3-1-1/6.2.8 but since most bridge beam webs are 'slender' the interaction between bending resistance and shear resistance must be evaluated in accordance with 3-1-5/7. No rules deal explicitly with combined bending, shear and torsion.

##### *Shear and torsion*

In an I section, the St Venant stresses are usually small enough to be neglected but if they do need to be considered (which might be the case for a long single unrestrained girder), the plastic shear resistance (to vertical bending) is reduced for coexistent St Venant shear stress - see 3-1-1/6.2.7(9). No reduction is required for warping shear stress in the web because it is nil in an I section.

In relation to shear buckling resistance, as there is no net vertical shear produced in a web of an I section from the circulatory St Venant torsional shear stresses, their

presence does not actually promote overall shear buckling. The effect of St Venant shear is more akin to an increased equivalent geometric imperfection in the plate, which would reduce the shear buckling resistance. However, St Venant shear stresses are generally very small and even this effect may be neglected for bridge girders.

The warping shear in the web due to the action of the compound section (paired beams with plan bracing in the bare steel stage, beams with slab in the composite stage) is automatically included in the global analysis. No additional interaction with torsion needs to be considered.

### **Bending and torsion**

In single beams, paired beams and composite beams subject to torsion, the girder flanges are subject to lateral bending and this needs to be taken into account when evaluating bending and buckling resistances.

With a single beam, torsion leads to warping moments in the two flanges and a (usually modest) moment in the cross section about the minor axis as a result of twist about the longitudinal axis. With paired beams, and with beams acting as part of a composite section, the lateral bending of the flanges is much reduced: the warping moments in the flanges due to global bending and torsion between supports will probably be negligible but there will be local warping effects due to torsional effects between bracing positions that does need to be taken into account.

Where elastic resistance of the cross section is to be verified, the criterion in 3-1-1/6.2.9.2 may be used, modified as follows:

$$\frac{M_{y,Ed}}{M_{y,el,Rd}} + \frac{M_{z,Ed}}{M_{z,el,Rd}} + \frac{M_{w,Ed}}{M_{el,fl,Rd}} \leq 1$$

Where:

- $M_{y,Ed}$  is the bending moment about the major axis
- $M_{y,el,Ed}$  is the elastic resistance for bending about the major axis
- $M_{w,Ed}$  is the warping moment in one flange
- $M_{el,fl,Rd}$  is the elastic bending resistance of the flange about the z-z axis
- $M_{z,Ed}$  is the minor axis bending moment due to the rotation of the section  
(=  $M_{y,Ed} \sin \phi$ , where  $\phi$  is the rotation at the cross section)
- $M_{z,el,Ed}$  is the elastic resistance for bending about the minor axis.

It is not necessary to include the effects of warping shear stress in the flange as its value is zero at the tips of the flange.

Where plastic interaction is to be verified, the criterion in 3-1-1/expression (6.41) may be adapted. Assuming that there is no significant axial force on the beam, the criterion is:

$$\left[ \frac{M_{y,Ed}}{M_{pl,y,Rd}} \right]^2 + \frac{M_{z,Ed}}{M_{pl,z,Rd}} + \frac{M_{w,Ed}}{M_{pl,fl,Rd}} \leq 1$$

Where:

$M_{pl,y,Rd}$  and  $M_{pl,z,Rd}$  are the plastic bending resistances of the beam about the y-y and z-z axes

$M_{pl,fl,Rd}$  is the plastic bending resistance of an individual flange, bending in its plane.

It would only be necessary to consider coexistent warping shear stress in the flange (which would reduce the plastic resistance moments) if it exceeds 50% of the plastic shear resistance of the flange; this situation does not arise with normal bridge configurations.

### **Bending and shear**

Shear-moment interaction should generally be verified as for a straight beam. As noted above, the shear buckling resistance does not need to be reduced for torsional effects but the resistance moment  $M_{f,Rd}$  in 3-1-5/7.1 (the product of the lever arm between flanges and the lesser of the axial resistances of the two flanges) should be reduced for the coexisting warping moments. A reduction factor for axial force is given in 3-1-5/5.4(2) and this factor can be modified to take account of warping moments, as follows:

$$\text{Factor} = \left( 1 - \frac{N_{Ed}}{(N_{bf,Rd} + N_{tf,Rd})} - \frac{M_{w,Ed}}{M_{fl,Rd}} \right)$$

where  $M_{w,Ed}$  is defined above,  $M_{fl,Rd}$  is either  $M_{el,fl,Rd}$  or  $M_{pl,fl,Rd}$  as appropriate, and  $N_{bf,Rd}$  and  $N_{tf,Rd}$  are the axial resistances of the two flanges.

## **4.2 Buckling resistance**

The rules in the Eurocodes for resistance to lateral torsional buckling have all been developed with the assumption that the girders are straight. Generally, these rules can be applied to curved beams, provided that the length of the beam around the curve is used, rather than the straight-line distance between supports, and that the effects of torsion (lateral bending of the flanges) are taken into account in interaction criteria.

The following guidance describes how the effects of torsion and warping moments can be taken into account when using the rules in the Eurocodes.

### **4.2.1 Single girders**

There is no guidance in BS EN 1993-1-1 on the effect of torsion on resistance to lateral torsional buckling but this omission has been addressed in BS EN 1993-6 (concerned with crane supporting structures). In its Annex A it gives a criterion in which the torsional effect and resistance are expressed as the bimoment but it is perhaps more helpful to re-express the criterion as:

$$\frac{M_{y,Ed}}{\chi_{LT} M_{y,Rk} / \gamma_{M1}} + \frac{C_{mz} M_{z,Ed}}{M_{z,Rk} / \gamma_{M1}} + \frac{k_w k_{zw} k_\alpha M_{w,Ed}}{M_{w,Rk} / \gamma_{M1}} \leq 1$$

in which:

$C_{mz}$  is the equivalent uniform moment factor for bending about the z-axis according to EN 1993-1-1 Table B.3. (For a simply supported beam with a parabolic bending moment diagram due to UDL  $C_{mz} = 0.95$ ; for a triangular bending moment diagram due to a single point load  $C_{mz} = 0.9$ )

$$k_w = 0.7 - 0.2 M_{w,Ed} / M_{w,Rd}$$

$$k_{zw} = 1 - M_{z,Ed} / M_{z,Rd}$$

$$k_\alpha = 1 / [1 - M_{y,Ed} / M_{cr}]$$

$M_{cr}$  is the elastic critical moment about the y-y axis

$M_{w,Ed}$  is the warping moment in one flange

$M_{w,Rk}$  is the characteristic bending resistance of the (weaker) flange.

$k_w$  can conservatively be taken as 0.7;  $C_{mz}$  and  $k_{zw}$  can conservatively be taken as 1; but  $k_\alpha$  does need to be evaluated.

Although flanges would normally be of such proportions that  $M_{w,Rk}$  could be taken as the plastic resistance moment, it would be prudent to use only the elastic resistance moment, since any tendency to utilize plastic deformation would result in increased lateral displacement and greater twist, neither of which would be recognized within the criterion.

For real situations (single curved girders at the erection stage), it is likely that both  $M_{y,Ed}$  (due to self weight) and  $\chi_{LT} M_{y,Rk}$  would be small and although the above criterion might be satisfied, the twist might be excessive, causing construction problems.

(Also, the rotation of the cross section would lead to minor axis bending, since

$$M_{z,Ed} = M_{y,Ed} \times \sin \phi.)$$

If buckling analysis software is available, a more direct approach to verification is to determine effects and elastic critical load using an FE model and to design in accordance with 3-1-1/6.3.4 (implemented by 3-2/6.3.4.1). If the loading is only self weight, no allowance for destabilizing effect need be made but if the load is applied at top flange level, some allowance for the destabilizing effect of the load should be made (see discussion in ED006<sup>[4]</sup>).

## 4.2.2 Paired girders

No guidance is given in BS EN 1993-1-1 for the resistance to LTB of paired girders without plan bracing. Either an elastic buckling analysis must be carried out or empirical rules such as those in SCI publication P356 and PD 6695-2 must be used. Guidance is given in P356.

From the elastic buckling analysis, or from the empirical rules, the non-dimensional slenderness may be determined (see advice in ED006) and, from that, the value of the reduction factor  $\chi_{LT}$  and thence the buckling resistance moment for each of the girders in the pair. The interaction criterion in Section 4.2.1 may then be applied.

For this situation, the value of  $M_{w,Ed}$  will change sign from the bracing position to midway between bracing positions. The value at the bracing position will usually be

numerically greater and should be used in the criterion. The value of  $M_{z,Ed}$  will often be small enough to be neglected.

### 4.2.3 Composite girders

In the configuration with the slab on top of the beams, the buckling resistance only needs to be verified in the regions adjacent to intermediate supports. Guidance on the use of rules in BS EN 1993-2 for this situation is given in P356 and the following interaction criterion for combined axial force and bending moment is suggested (there will be some modest axial force due to interaction between beams related to unequal loading on each of the beams and there may be axial force due to soil pressure in an integral bridge):

$$\frac{N_{Ed}}{N_{b,Rd}} + \frac{M_{Ed}}{M_{b,Rd}} \leq 1$$

Where there are additional effects due to lateral bending of the flange these can be introduced in the same way as for single beams according to BS EN 1993-6.

The criterion then becomes:

$$\frac{N_{Ed}}{N_{b,Rd}} + \frac{M_{Ed}}{M_{b,Rd}} + \frac{k_w k_{zw} k_\alpha M_{w,Ed}}{M_{w,Rk} / \gamma_{M1}} \leq 1$$

in which:  $k_w$ ,  $k_{zw}$  and  $k_\alpha$  are as defined in Section 4.2.1 and  $k_{zw}$  is usually taken as 1.0.

Where the simplified method of verification in 3-2/6.3.4.2 is used (considering the lateral buckling of a Tee section comprising the compression flange and compression zone of the web),  $N_{b,Rd}$  and  $M_{b,Rd}$  can be evaluated by that method and  $M_{cr}$  (for LTB) can be derived from the value of  $\bar{\lambda}_{LT}$ . Guidance on the simplified strut method is given in P356.

## 4.3 Fatigue assessment

In accordance with usual practice, fatigue design stresses need to be determined by elastic analysis. The effects of lateral bending of the bottom flanges need to be included in that determination. The fatigue details that will experience enhanced stress due to lateral bending effects are clearly those where there are attachments to the flange (stiffeners and bearing plates), transverse butt welds and bolted splices.





# SERVICEABILITY CONSIDERATIONS

The most commonly experienced criterion at the serviceability limit state (SLS) is the limitation of elastically determined stresses to the yield strength of steel, although this usually only arises when plastic resistance is utilized at ultimate limit state. In the case of curved girders, the effects include those due to warping but no special requirements apply.

Limits on deflection at SLS are sometimes imposed, such as when the soffit is nominally flat or when a clearance gauge might be infringed but these apply to the completed structure and again no special requirements apply for curved girders.

However, because single curved girders and paired curved girders twist (under self weight in the first case, under the weight of wet concrete in the second) it may be appropriate to impose limits on the twist. The two situations are discussed below.

## 5.1 Twist of single curved girders

Where the proposed construction method involves the erection of individual curved girders (typically in ladder deck construction), the girders will usually be very flexible torsionally and will experience significant twist under their own self weight. Excessive twist would cause problems in connecting components (such as cross girders) and would lead to significant bending about the minor axis of the girder. The designer should consider carefully the consequences of rotation and the sensitivity of the structure to values greater than those calculated and should ensure that the rotations are within acceptable limits for construction.

The of twist in a (relatively) long curved girder can only be determined using a computer model (such as a 3D FE model) that can fully represent the interaction between warping torsion and St Venant torsion.

If it is found that the twist would be excessive, then the construction method would need to be modified (for example by introducing a temporary support at mid-span).

Since the permanent works designer is responsible for setting out a proposed construction method, it is his responsibility, in the first place, to calculate and determine the acceptability of the twist at this construction stage. Also, since the predicted rotations will be of importance to the constructor, their values should be communicated as part of the design information that is provided to the constructor.

## 5.2 Twist of paired curved girders

The twist of paired girders under the weight of wet concrete may also give rise to problems if it is excessive, though this is mainly of concern for multi-girder decks.

The torque arising from the curvature will be resisted mainly by differential bending of the two girders rather than by torsional moments in the girders and the angle of twist will generally be much smaller than that likely to be experienced with single curved girders. However, if not properly allowed for, any twist would result in either an incorrect finished surface of the slab (if it were cast to a specified thickness above the girders) or an unequal, and possibly excessive, thickness of slab (if cast to a surface profile).

The permanent works design should therefore determine the deflections and rotations for the proposed construction method and ensure that these are within acceptable limits. (In choosing limits, it should be noted that twist is difficult to predict accurately and the designer should also consider the consequences should the actual displacements prove excessive – would corrective measures be necessary and if so how might they be achieved?)

Deflections and rotations can be determined from a grillage analysis but it should also be noted that the elastic critical load at this stage may be fairly low and consequently the deflections might be magnified by second order effects.

The deflections and twists at this stage should be provided to the constructor as part of the design information.

In multi-girder bridges, with several pairs of main girders, the spacing between pairs is a key consideration for the support of the slab formwork, particularly when precast units are used, but the spacing may be affected by the twists of adjacent pairs. The provision of control bracing, typically of simple angle members between the pairs, is especially important for curved decks.





# CONSTRUCTION ASPECTS

## 6.1 Fabrication

Fabrication of bridge girders curved in plan does not impose any major problems for the steelwork contractor, though some aspects require a little more attention.

The cutting of flanges from plate is now normally carried out by numerically controlled machines, using data from computerised solid models. Curvature, even non-uniform curvature, is thus easily achieved. However, it should be appreciated that there will be a little more wastage and that in some cases it will be necessary to cut flanges one at a time, rather than cutting several at the same time (using multiple head machines). This will lead to slightly higher costs than for straight girders.

I section girders are now frequently welded in semi-automatic welding machines (often referred to as T and I machines). These machines can handle a certain amount of plan curvature, depending on the size of the machine; extra attention is required to feed the girder over the rollers and through the welding head. Some fabricators will find it necessary to assemble and weld the sections in jigs, rather than use a T and I machine. Again, the effect of the curvature is to increase costs slightly.

Welding of transverse web stiffeners takes place after the I section has been formed. Curvature has no effect on this process, even when robotic equipment is used.

## 6.2 Transport

The effect of plan curvature is to increase the overall width of girder sections and this can lead to transportation problems, either in transporting single girders or paired girders.

For reasons discussed below, the pairing of girders in multiple girder bridges is especially advantageous for curved girders. Pairs of girders typically have an overall width exceeding 4 m and when they are curved in plan the width may approach the 5 m limit above which notification must be given to the police (see guidance on transport limits in P185, GN7.06<sup>[5]</sup>).

The overall width for transportation is not simply the width of the rectangular 'box' into which the girders could theoretically just be fitted. A girder (or girder pair) is usually supported at one end (adjacent to the tractor unit) and at perhaps  $\frac{2}{3}$  of its length. The cantilever behind the rear bogies will then also extend sideways.

### 6.3 Erection

The initial complication for erection of a curved girder is in ensuring that it can be lifted from above its centre of gravity. With a straight girder the CG and the lifting lugs (or centres of lifting straps) all lie on the girder centre-line. With a (single) curved girder it can be difficult to arrange that the line between lifting points passes through the CG. Extra provisions may be needed to ensure that the girder will hang vertically when lifted. Additionally, the ends of a curved girder will twist when it is lifted (lifting points are typically at the quarter points of the girder length), which can complicate the connection to a previously erected girder.

For multi-girder bridges, it is preferable to lift girders in braced pairs, to minimise twist after erection. The connection of pairs of girders usually needs to take place before transportation, as there is unlikely to be any suitable facility to do so on site (unless the site is large and well equipped for steelwork assembly).

For ladder deck bridges, girders can only be transported and erected singly (except where preassembly on site is possible and very large cranes are available). The long and heavy girders will twist significantly before the cross girders are connected and it may be necessary to provide a temporary support at mid-span to reduce the twist.

Generally, the procedures for lifting and connecting curved girders will need close attention, for safety and to ensure an efficient process. Eccentric effects associated with curved girder geometry may result in unstable conditions until the superstructure steelwork is complete. The stability of incomplete steelwork requires attention at all stages.

### 6.4 Slab construction

As noted in Section 5.2, braced curved girders will twist slightly under the weight of wet concrete, in the same way as straight paired girders when the loading on each is not equal. The vertical bending stiffness of the main girders will provide the principal resistance to twist but the effects do need to be taken into account to ensure that an appropriate preset can be provided.

Where there is a large cross-fall to the slab (to provide super-elevation in the carriageway) the slab will vary significantly in thickness across the width of the girder flanges (which will normally be horizontal transversely) and thus between girders as well. The reinforcement must be detailed to suit the variation.





# ARTICULATION

## 7.1 Thermal effects

The dominant thermal effect for the choice of lateral restraints at bearings is the uniform temperature increase/decrease, given by BS EN 1991-1-5 and its National Annex. For a relatively narrow straight bridge, this results in a significant change in length but very little change in width. It is therefore usual to select a fixed point, either at one end or at a central support and to provide one longitudinally guided bearing at each of the other supports; transverse expansion/contraction at each support is small and is unlikely to cause any problems at the expansion joints.

BS EN 1991-1-5 does give values for a temperature difference across the width of a bridge and this action would cause plan bending of the deck. Plan bending would be restrained by the guided bearings but the effects are usually small.

When the bridge deck is curved in plan, the accommodation of expansion and contraction becomes a little more complex; a simple linear articulation is not possible. There are three basic articulation arrangements for bridges curved in plan and these are discussed below.

## 7.2 Radial guiding

With this option, the bridge deck is constrained to expand/contract freely in a radial direction from a fixed point as a result of uniform temperature. The 'lateral' restraint at each support (against the effects of horizontal loads and wind action) is thus not orthogonal to the bridge axis but the longitudinal component of the restraint force will have negligible effect on design of the beams. This articulation arrangement is shown in Figure 7.1.

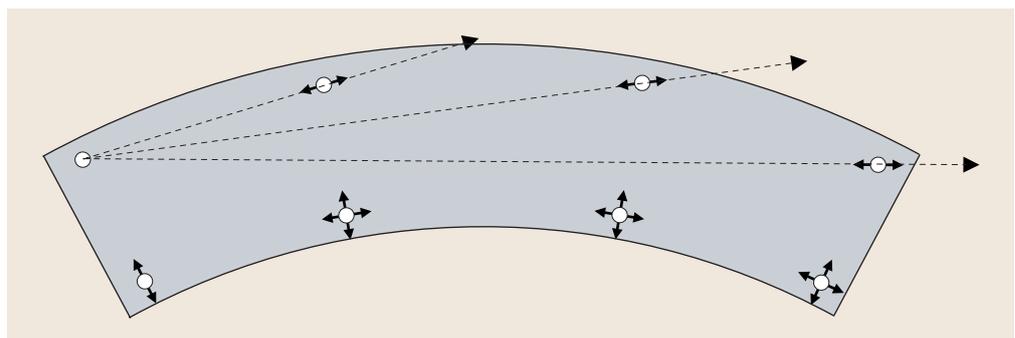


Figure 7.1  
Radially guided  
articulation

Two consequences of selecting this arrangement are obvious: care will be needed in installing all the guided bearings to ensure that the intended alignment is achieved; the movement at the expansion joint is not perpendicular to the joint (unless the joint is normal to a radial line and skew to the bridge axis). The capacity of the joint for these combined longitudinal and transverse movements should be verified.

This articulation arrangement is commonly used.

The effects of horizontal temperature difference are similar to those on a straight deck and can usually be accommodated in the same way.

### 7.3 Radial guiding with plan rotation

To overcome the problem with radial guiding of movement at the expansion joint not being perpendicular to the joint, the guided bearing at that support can be arranged to guide movement perpendicular to the joint. As a result, the deck will rotate in plan as it expands and contracts and the guided bearings at intermediate supports will need to be aligned to suit. The articulation arrangement is shown in Figure 7.2.

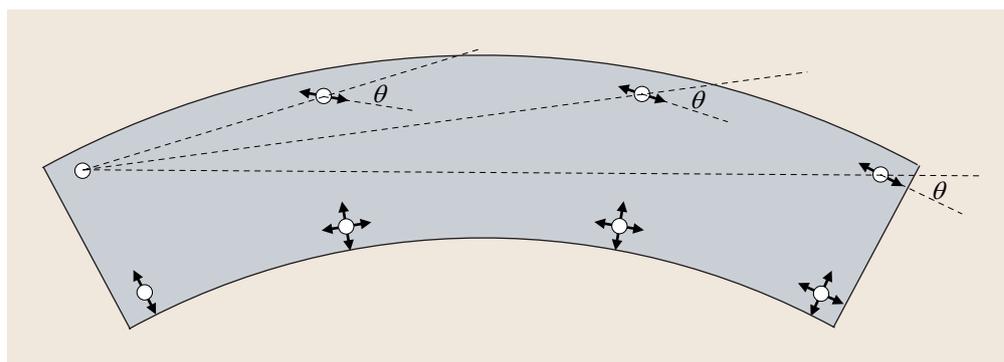


Figure 7.2  
Radial guiding with  
plan rotation

This arrangement still requires careful alignment of all the guided bearings but, as can be seen from the Figure, the required alignment is easily defined.

Again, the effects of horizontal temperature difference will be modest, for most bridges, and can be accommodated by slack in the bearings.

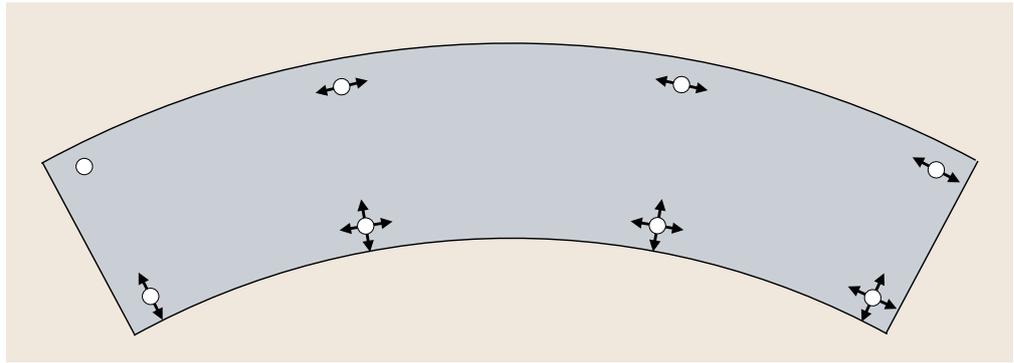
This arrangement is suitable for wide decks, small radius curves and bridges where the radius changes along the length of the bridge, although it has not often been used.

### 7.4 Guiding around the curve

This apparently simple articulation arrangement aligns all the guided bearings to guide movements parallel to the bridge axis at each support; it is an arrangement that is often chosen for longer bridges. The expansion/contraction is 'guided around the curve', as shown in Figure 7.3.

However, due consideration of the effect of a uniform temperature change in the deck will show that, without restraint, the curved deck will change its radius as well

Figure 7.3  
Guiding around  
the curve



as increasing in length. The effect of the guided restraints along the original curve is to constrain that change and to bend the deck in plan, constraining it to the original radius. The bending moment in plan required to constrain the deck in this way may be determined as follows:

Consider a deck with a circular curve in plan of radius  $R$ . For a temperature rise of  $T$  the change in curvature, if unconstrained, is given by:

$$\Delta\psi = \frac{\alpha \times T}{R}$$

where  $\alpha$  is the coefficient of expansion.

In practice, the change in radius is quite small; for example, for a 28 m/40 m/28 m span bridge at 150 m radius, the 'lateral' movement at an intermediate support, if unrestrained, would be only about 1 mm.

To prevent this change in curvature requires a bending moment given by:

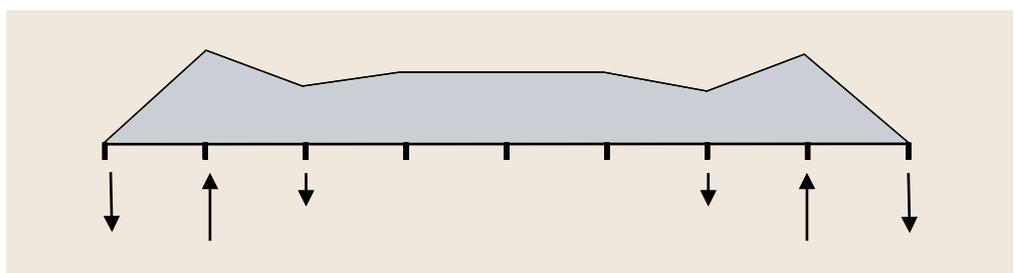
$$M = EI\Delta\psi = \frac{EI\alpha T}{R}$$

where  $E$  is the modulus of elasticity and  $I$  is the second moment of area of the whole deck for horizontal bending.

In plan the deck is a multi-span continuous beam and the restraint moment is achieved mainly by opposing horizontal forces at the supports, as shown for a multi-span viaduct in Figure 7.4.

The magnitude of the forces depends on the span arrangement and the flexibility at supports (including flexibility of restraint bracing systems) but with equal spans the

Figure 7.4  
Plan bending due  
to guiding around  
the curve



force at the abutment may be taken to be approximately  $1.5M/L$ , where  $L$  is the length of the end span.

Lateral forces due to this constraint can be very large if the deck is wide or the radius is small. However, the flexibility of the support bracing will reduce the value. It is essential that an appropriate evaluation of the 3D behaviour of the structure is undertaken taking account of all flexibilities, to establish the restraint forces on the bearings.

If the curvature of the bridge deck is not uniform, this may tend to create a natural 'null point' and the interaction with a fixed bearing, if provided at one end, would need to be considered carefully.

## **7.5 Effects in integral bridges**

Curved bridges can be designed as integral bridges, provided that the additional effects are taken into account.

If the two abutments are square to a line between them, the additional effect is principally plan bending of the deck (moment = force times offset).

If the abutments are square to the centreline at the ends, then the normal pressures on the two endscreen walls will have components of force perpendicular to the line between them. For a single span bridge, there is no means to resist such pressures so they must be accompanied by shearing stresses. For a multiple span bridge, the normal pressures result in lateral forces on intermediate supports, plus some shearing stresses: the soil-structure interaction must be modelled in order to determine the values of the forces.





# CHOOSING BRACING INTERVAL

The spacing of the bracing affects the lateral bending of the flanges and the forces in the bracing.

In a ladder deck bridge, the spacing of the cross girders is dictated by the design of the deck slab and is usually no greater than 4 m. This will give rise to bending stresses due to curvature of no more than about 10% of the total stress due to vertical bending.

In a multi-girder bridge that is straight in plan, bracing is typically located at a distance from the intermediate support of about 10 times the flange width - this ensures that the reduction factor for buckling is not significantly below unity and thus the design is economic. Bracing in the rest of the span is more widely spaced.

In a multi-girder bridge that is curved in plan, a uniform spacing of bracing is often adopted in the span. For economy, the stress due to lateral bending (which is greatest at mid-span and at intermediate supports) should be limited to about 10% of the design value of yield strength, if an elastic criterion applies, or about 20% if a plastic criterion applies (i.e. for tension in the bottom flange at mid-span). The following table may be used as a guide to the magnitude of elastic stress due to lateral bending for different bracing intervals (values in excess of 30% are not shown, as that is unlikely to be a practical situation).

SPACING OF BRACING $L$ (m)	LATERAL BENDING STRESS (% OF VERTICAL BENDING STRESS) FOR PLAN RADIUS $R$ (m)			
	180	300	500	800
<i>500 mm wide flange</i>				
5.0	17%	10%	6%	4%
6.5	28%	17%	10%	6%
8.0		26%	15%	10%
<i>600 mm wide flange</i>				
6.0	20%	12%	7%	5%
8.0		21%	13%	8%
10.0			20%	13%
<i>800 mm wide flange</i>				
8.0	27%	16%	10%	6%
10.0		25%	15%	9%
12.0			22%	14%

Table 8.1  
Lateral bending stress as a function of bracing interval

Note: Values based on assumption of uniform bending stress over the bracing interval and a lateral bending moment of  $FL^2/10R$ , where  $F$  is the force in the flange.



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# CREDITS



**Cover & 3** Hunslet viaduct.  
Photo courtesy of Tata Steel.



**3** Highfield Lane bridge.  
Photo courtesy of Rotherham MBC.



**4** M25/A2 interchange bridge.  
Photo courtesy of Mabey Bridge Ltd.



**4** Bargoed Regeneration Scheme.  
Courtesy of Hochtief (UK) Construction Ltd. Photo by Cloud 9.



**5** M50 overbridge, Dublin.  
Photo courtesy of National Roads Authority, Rol.



# APPENDIX A - WORKED EXAMPLES

Publication P357, Composite Highway Bridge Design: Worked Examples, has a detailed worked example of a 2-span multi-girder composite bridge that illustrates the process of setting out structural and loading details, analysis for the various construction and in-service stages, and verification of the design resistance in accordance with Eurocodes 3 and 4. The bridge is straight in plan. This Appendix considers the consequences on the design for a similar bridge that is curved in plan. It is not considered necessary to repeat the level of detailed evaluation of the various parameters (and thus no need to adopt the calculation sheet format used in P357) but instead comparisons are made between the design values of effects (bending moments, deflections, etc.) and the additional interactions (plan bending of the flanges) that need to be considered and verified are highlighted.

## A.1 Structural configuration

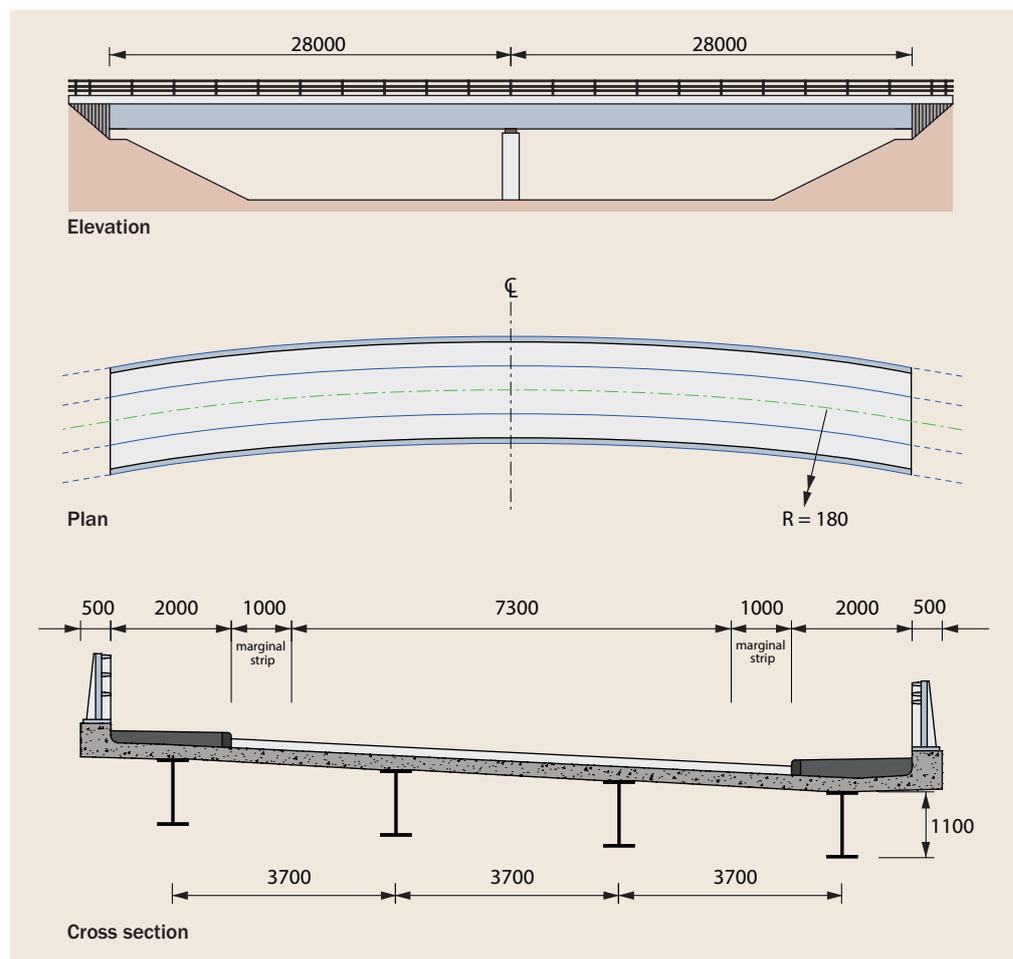


Figure A.1  
General arrangement

As in P357, the beams are assumed to be braced in pairs. Four intermediate planes of X bracing are provided in each span at equal spacing along the centreline of each pair, as shown in Figure A.2.

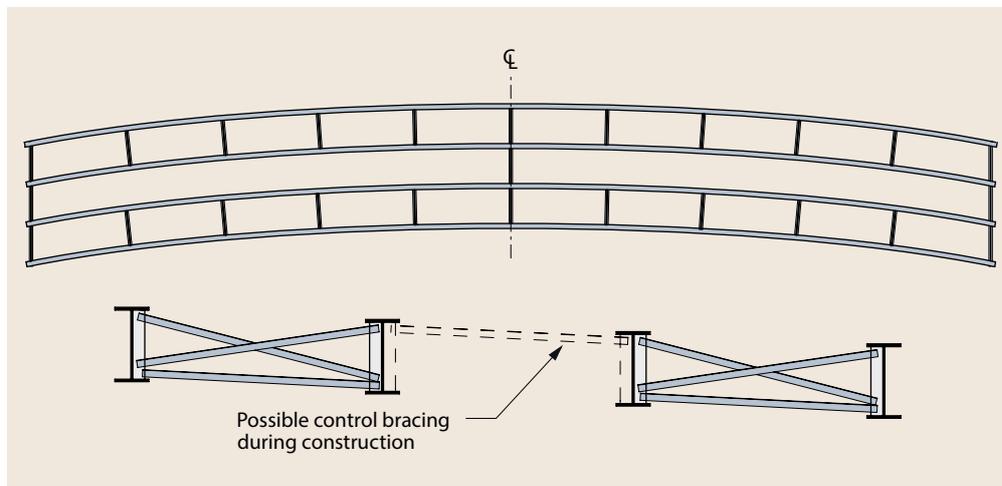


Figure A.2  
Intermediate Bracing

## A.2 Girder make-up and section properties

For ease of comparison, the same girder make-up is used as in P357, even though it might be found that a small increase in section size is needed.

	SPAN GIRDER (APPROX 22 m)	PIER GIRDER (APPROX 12 m)
Top flange	500 × 40	500 × 40
Web	10 (12 at abutment)	14
Bottom flange	500 × 40	600 × 60
Top rebars	B16 @ 150 mm crs	B25 @ 150 mm
Bottom rebars	B16 @ 150 mm crs	B25 @ 150 mm

The section properties are:

### Bare steel cross sections

		SPAN GIRDER	PIER GIRDER	
Area	$A$	50200	70000	(mm <sup>2</sup> )
Height of NA		550	436	(mm)
Second moment of area	$I_y$	$1.212 \times 10^{10}$	$1.562 \times 10^{10}$	(mm <sup>4</sup> )
Elastic modulus, centroid top flange	$W_{tf,y}$	$2.287 \times 10^7$	$2.425 \times 10^7$	(mm <sup>3</sup> )
Elastic modulus, centroid bottom flange	$W_{bf,y}$	$2.287 \times 10^7$	$3.847 \times 10^7$	(mm <sup>3</sup> )
Section class		4*	3 (hogging)	
Plastic bending resistance	$M_{pl}$	8237	9882	(kNm)

\* The section is only marginally Class 4. For stress build-up during construction the bare steel section may be treated as Class 3, since the final composite section is Class 3 or better.

### Composite cross sections in span girder (sagging)

		SHORT TERM ( $n_0 = 6.0$ )	LONG TERM ( $n_0 = 6.0$ )	
Area	$A$	209800	107500	(mm <sup>2</sup> )
Height of NA		1098	934	(mm)
Second moment of area	$I_y$	$3.288 \times 10^{10}$	$2.634 \times 10^{10}$	(mm <sup>4</sup> )
Elastic modulus, top of slab	$W_c$	6.534E+08	$9.439 \times 10^8$	(mm <sup>3</sup> )
Elastic modulus, centre of top flange	$W_{tf,y}$	1.827E+09	$1.804 \times 10^8$	(mm <sup>3</sup> )
Elastic modulus, centre of bottom flange	$W_{bf,y}$	$3.050 \times 10^7$	$2.882 \times 10^7$	(mm <sup>3</sup> )
Plastic bending resistance	$M_{pl}$	13070	107500	(kNm)

The cross section of the span girder is class 1, provided that the top flange is restrained by shear connectors within the spacing limits in 4-2/6.6.5.5 (in this case, max spacing 730 mm, max edge distance 299 mm).

### Composite cross sections in pier girder (hogging)

<b>CRACKED</b>				
Area	$A$	94250		(mm <sup>2</sup> )
Height of NA		653		(mm)
Second moment of area	$I_y$	$2.845 \times 10^{10}$		(mm <sup>4</sup> )
Elastic modulus, top rebars	$W$	$4.184 \times 10^7$		(mm <sup>3</sup> )
Elastic modulus, centre of top flange	$W_{tf,y}$	$6.663 \times 10^7$		(mm <sup>3</sup> )
Elastic modulus, centre of bottom flange	$W_{bf,y}$	$4.567 \times 10^7$		(mm <sup>3</sup> )
Section class		3		
Plastic bending resistance	$M_{pl}$	16990		(kNm)

For in-plane bending of the bottom flange, the elastic and plastic moduli of the individual flanges are:

		MID-SPAN (500 × 40 MM)	PIER (600 × 60 MM)
Elastic modulus	$W_{el,fl}$	$1.67 \times 10^6$	$3.6 \times 10^6$
Plastic modulus	$W_{pl,fl}$	$2.50 \times 10^6$	$5.4 \times 10^6$

## A.3 Actions

The permanent actions due to self weight of structural and non-structural elements are as given in P357 and are summarized below.

CHARACTERISTIC VALUE		
Steel density	77	kN/m <sup>3</sup>
Concrete density	25	kN/m <sup>3</sup>
Surfacing on carriageway	4.63	kN/m <sup>2</sup>
Footway	4.80	kN/m <sup>2</sup>
Parapet (on each side)	2.0	kN/m

Traffic loading is taken as Group 5 loading, combining LM1 and an abnormal SV100 vehicle. Loads are positioned to occupy the most onerous positions on the influence surface for the effect being considered.

Shrinkage effects need to be considered for maximum hogging moment at the intermediate pier. The secondary moments are determined for an effective restraint moment of 1388 kN over the uncracked regions, as in P357.

Thermal effects need to be considered for maximum sagging moment in mid-span. The secondary moments are determined for an effective restraint moment of 438 kNm (characteristic value) over the uncracked regions, as in P357.

## A.4 Global analysis

A 3D model similar to that used for the example in P357 was created. The model had FE plate elements for the deck slab and girder webs, and beam elements for the flanges. A similar mesh density was used for both the girders and the slab. A view of part of the model is shown in Figure A.3.

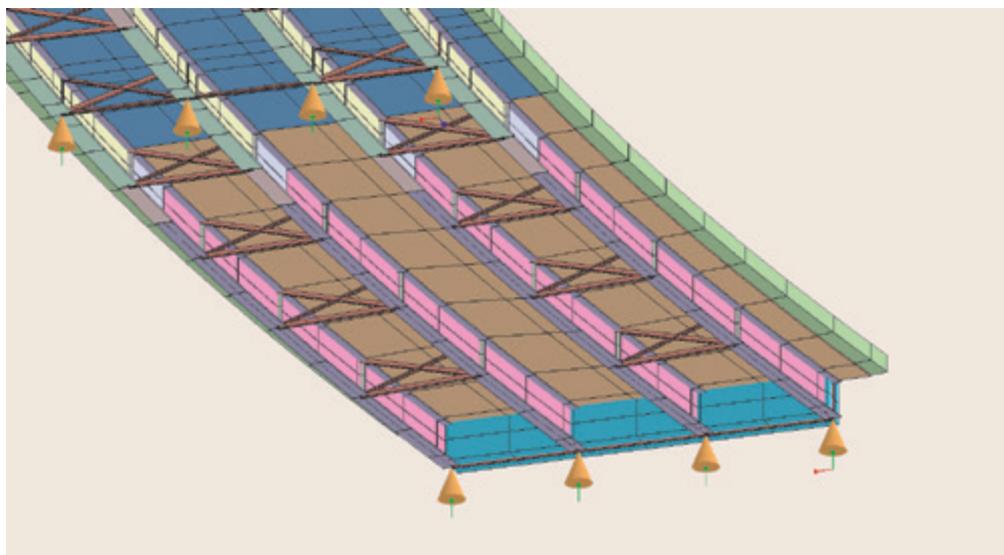


Figure A.3  
View of the underside  
of one span of the  
analysis model

The construction stages analyzed were the same as for the model in P357:

- Stage 1** All steelwork, wet concrete in span 1
- Stage 2** Composite structure in span 1 (long-term properties), wet concrete in span 2
- Stage 3** Composite structure in both spans (long-term properties)
- Stage 4** Composite structure (short-term properties).

## A.5 Analysis results

### A.5.1 Effects at ULS

As for the straight bridge, the most onerous (vertical bending) effects on the four identical girders occur on one of the inner girders; the values of the ULS bending moments in the four girders at each stage are given below in Table A.1 for the cross section at the pier and Table A.2 for the cross section at mid-span (actually, at the second brace from the abutment, approximately 11.2 m from the support). Values at the splice are not given, as the verification at that location is not considered in this example.

Table A.1  
ULS design values  
of vertical bending  
moment at pier  
(curved deck)

	BENDING MOMENT $M_y$ (kNm)			
	GIRDER 1	GIRDER 2	GIRDER 3	GIRDER 4
Stage 1 (wet concrete, span 1)	-1922	-2789	-2401	-2196
Stage 2 (wet concrete, span 2)	-1514	-3076	-2103	-1964
Stage 3 (surfacing etc.)	-2603	-1481	-1523	-2791
Shrinkage	-1802	-1213	-1213	-1493
Stage 4 (traffic) – max hogging, girder 2	-2214	-3637	-3113	-1571

Table A.2  
ULS design values  
of vertical bending  
moment at mid-span  
(curved deck)

	BENDING MOMENT $M_y$ (kNm)			
	GIRDER 1	GIRDER 2	GIRDER 3	GIRDER 4
Stage 1 (wet concrete, span 1)	1828	3681	2633	2915
Stage 2 (wet concrete, span 2)	-1250	-1464	-1717	-1730
Stage 3 (surfacing etc.)	1607	1185	1126	1800
Stage 4 (traffic) – max sagging, girder 2	3445	4946	4558	3270
Temp diff	119	151	167	139

For comparison, the effects on the inner girder of the straight bridge are presented in Table A.3 and Table A.4. The increases in total design moment on the most heavily loaded girder (girder 2) are 10% at the pier and 20% in mid-span.

Table A.3  
ULS design values  
of vertical bending  
moment at pier  
(straight deck)

	BENDING MOMENT $M_y$ (kNm)			
	GIRDER 1	GIRDER 2	GIRDER 3	GIRDER 4
Stage 1 (wet concrete, span 1)	-2052	-2573	-2573	-2052
Stage 2 (wet concrete, span 2)	-1700	-2499	-2499	-1700
Stage 3 (surfacing etc.)	-2212	-1705	-1705	-2212
Shrinkage	-1383	-1552	-1552	-1383
Stage 4 (traffic) – max hogging, girder 3	-1226	-2850	-3621	-2435

Table A.4  
ULS design values  
of vertical bending  
moment at mid-span  
(straight deck)

	BENDING MOMENT $M_v$ (kNm)			
	GIRDER 1	GIRDER 2	GIRDER 3	GIRDER 4
Stage 1 (wet concrete, span 1)	2381	3132	3132	2381
Stage 2 (wet concrete, span 2)	-1479	-1714	-1714	-1479
Stage 3 (surfacing etc.)	1448	1266	1266	1448
Stage 4 (traffic) – max sagging, girder 3	2204	4213	4952	3582
Temp diff	216	200	200	216

In addition to the effects of vertical bending, the flanges of the girders are subjected to plan bending as a result of these loads. In the global model, which has nodes only at bracing positions and midway between, the bending moment diagrams do not give a sufficient picture of the plan bending behaviour. This is discussed further below.

### A.5.2 Effects at SLS

For the curved bridge, the only SLS effects that will be considered are the deflections of the girders during construction. Of particular interest are the relative deflections of the two pairs of braced girders at mid-span. These values were not given for the straight bridge in P357 but values have been extracted from that earlier analysis, for comparison. Deflections are given in Table A.5 and Table A.6.

Table A.5  
SLS deflections of  
girders at mid-span  
(curved deck)

	DEFLECTION OF TOP/BOTTOM FLANGES (mm)			
	GIRDER 1	GIRDER 2	GIRDER 3	GIRDER 4
Stage 1 (wet concrete, span 1)	41	77	54	58
Stage 2 (wet concrete, span 2)	-14	-16	-18	-19
Stage 3 (surfacing etc.)	13	12	12	15
Total permanent deflection	40	73	48	54

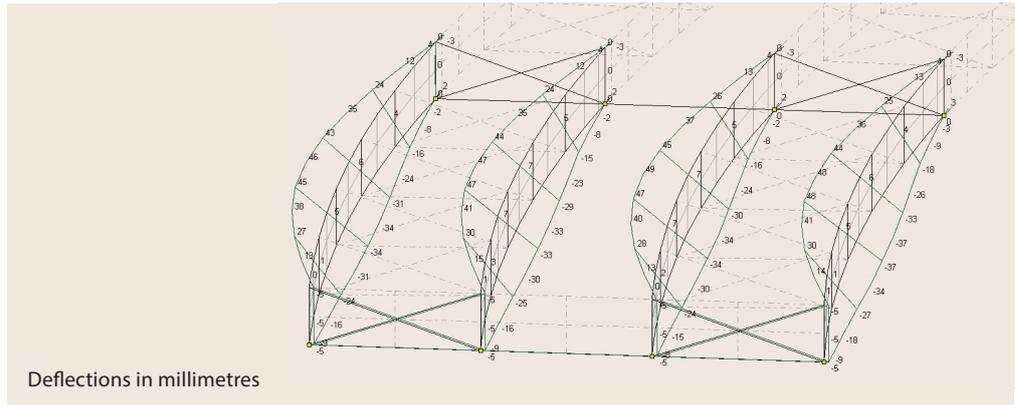
Table A.6  
SLS deflections of  
girders at mid-span  
(straight deck)

	DEFLECTION OF TOP/BOTTOM FLANGES (mm)			
	GIRDER 1	GIRDER 2	GIRDER 3	GIRDER 4
Stage 1 (wet concrete, span 1)	47	62	62	47
Stage 2 (wet concrete, span 2)	-19	-21	-21	-19
Stage 3 (surfacing etc.)	15	13	13	15
Total permanent deflection	43	54	54	43

Additionally, although it would be normal practice to lift pairs of braced beams, rather than individual girders, the deflections of individual unbraced girders in the curved bridge have been determined. The deflections are presented graphically in Figure A.4, for the four girders over a single span (ignoring the effect of the short cantilever into the adjacent span).

The maximum twist occurs on the innermost girder, with a relative deflection between top and bottom nodes of 85 mm. This is equivalent to a rotation of 4.1° (the nodes are 1200 mm apart vertically).

Figure A.4  
Horizontal deflections  
of unbraced girders  
under self weight  
(SLS values)



## A.6 The effects of combined actions

The following Tables present the total effects of the combined actions at the pier and mid-span, in terms of total bending moment and shear and stresses in the flanges. The effects of the very small axial effects are not tabulated.

	BOTTOM FLANGE		TOP FLANGE		TOP REBAR			
	$M_y$ (kNm)	$F_z$ (kN)	$W$ ( $10^6 \text{ mm}^3$ )	$\sigma$ (N/mm <sup>2</sup> )	$W$ ( $10^6 \text{ mm}^3$ )	$\sigma$ (N/mm <sup>2</sup> )	$W$ ( $10^6 \text{ mm}^3$ )	$\sigma$ (N/mm <sup>2</sup> )
Stage 1	-2789	692	38.47	-72	24.25	115		
Stage 2	-3076	-20	45.67	-67	66.63	46	41.84	74
Stage 3	-1481	260	45.67	-32	66.63	22	41.84	35
Shrinkage ( $\gamma_{sh} = 1$ )	-1213	21	45.67	-27	66.63	18	41.84	29
Stage 4 (traffic)	-3637	431	45.67	-80	66.63	55	41.84	87
	-12196	1384		-278		256		225

Table A.7  
Design effects at pier  
(curved deck)

Note: Total axial force = 538 kN (compression) and the stress in the steel section is 6 N/mm<sup>2</sup>.

	BOTTOM FLANGE		TOP FLANGE		TOP OF SLAB			
	$M_y$ (kNm)	$F_z$ (kN)	$W$ ( $10^6 \text{ mm}^3$ )	$\sigma$ (N/mm <sup>2</sup> )	$W$ ( $10^6 \text{ mm}^3$ )	$\sigma$ (N/mm <sup>2</sup> )	$W$ ( $10^6 \text{ mm}^3$ )	$\sigma$ (N/mm <sup>2</sup> )
Stage 1	3681	-35	22.87	161	22.87	-161		
Stage 2	-1464	107	28.82	-51	180.4	8	943.9	1.6
Stage 3	1185	10	28.82	41	180.4	-7	943.9	-1.3
Stage 4 (traffic)	4946	2	30.50	162	1827	3	653.4	-7.6
Temp diff	151	-12	30.50	5	1827	0	653.4	-0.2
	8499	72		318		-157		-7.5

Table A.8  
Design effects  
at mid-span  
(curved deck)

Note: Total axial force = 879 kN (tension) and the stress in the steel section is 5 N/mm<sup>2</sup>.

	BOTTOM FLANGE				TOP FLANGE		TOP REBAR	
	$M_y$ (kNm)	$F_z$ (kN)	$W$ (10 <sup>6</sup> mm <sup>3</sup> )	$\sigma$ (N/mm <sup>2</sup> )	$W$ (10 <sup>6</sup> mm <sup>3</sup> )	$\sigma$ (N/mm <sup>2</sup> )	$W$ (10 <sup>6</sup> mm <sup>3</sup> )	$\sigma$ (N/mm <sup>2</sup> )
Stage 1	-2573	689	38.47	-67	24.25	106		
Stage 2	-2499	-13	45.67	-55	66.63	38	41.84	60
Stage 3	-1705	308	45.67	-37	66.63	26	41.84	41
Shrinkage ( $\gamma_{sh} = 1$ )	-1552	-45	45.67	-34	66.63	23	41.84	37
Stage 4 (traffic)	-3621	499	45.67	-79	66.63	54	41.84	87
	-11950	1438		-272		247		225

Table A.9  
Design effects at pier  
(straight deck)

	BOTTOM FLANGE				TOP FLANGE		TOP OF SLAB	
	$M_y$ (kNm)	$F_z$ (kN)	$W$ (10 <sup>6</sup> mm <sup>3</sup> )	$\sigma$ (N/mm <sup>2</sup> )	$W$ (10 <sup>6</sup> mm <sup>3</sup> )	$\sigma$ (N/mm <sup>2</sup> )	$W$ (10 <sup>6</sup> mm <sup>3</sup> )	$\sigma$ (N/mm <sup>2</sup> )
Stage 1	3132	-3	22.87	137	22.87	-137		
Stage 2	-1714	-5	28.82	-59	180.4	10	943.9	1.8
Stage 3	1265	114	28.82	44	180.4	-7	943.9	-1.3
Stage 4 (traffic)	4952	-692	30.50	162	1827	3	653.4	-7.6
Temp diff	200	25	30.50	7	1827	0	653.4	-0.3
	7835	-561		291		-131		-7.4

Table A.10  
Design effects  
at mid-span  
(straight deck)

## A.7 Discussion of load sharing between girders

At the wet concrete stage, the effect of the curvature was to increase the bending moments in the girder further from the centre in each pair, both in mid-span and at the intermediate support, making the second girder (from the centre) the most heavily loaded. Under the surfacing load, the change in load distribution became a little more complex, increasing the bending moment in mid-span and decreasing it at the support. Under traffic loading, the maximum moments in the second girder were similar to those under an inner girder in the straight deck. The overall result was an increase in design moment of about 2% at the pier and 8% at mid-span. Bending moments for all the girders at Stage 1 and Stage 4 for maximum sagging are shown in Figure A.5 and Figure A.6.

## A.8 Plan bending of bottom flange

The global analysis model is rather coarse for determining the lateral bending of the bottom flange; it has nodes only at bracing positions and midway between those positions. The lateral bending moments in the bottom flange as a result of the 'kinked' profile (a series of chords in plan) are shown in Figure A.7. The values are total effects (due to construction plus traffic loads), for the situations for maximum hogging and sagging moments in Girder 2.

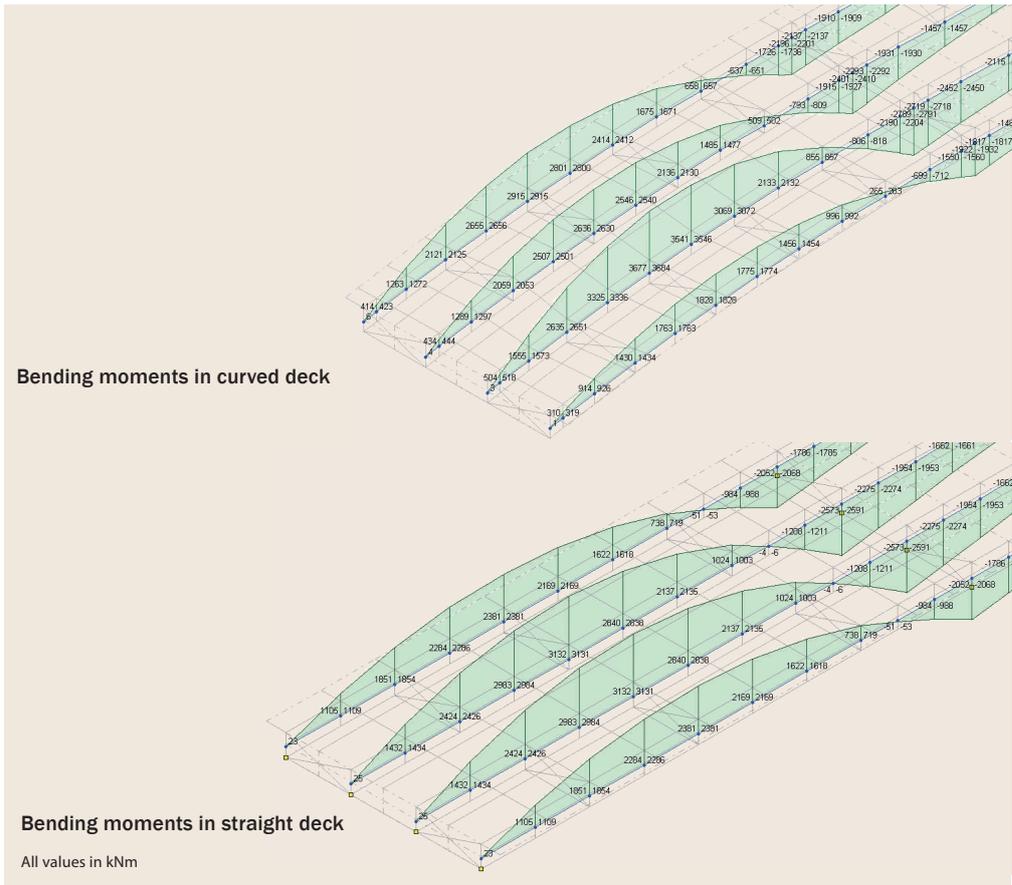


Figure A.5  
Comparison of  
bending moments in  
girders at Stage 1 –  
wet concrete in span 1

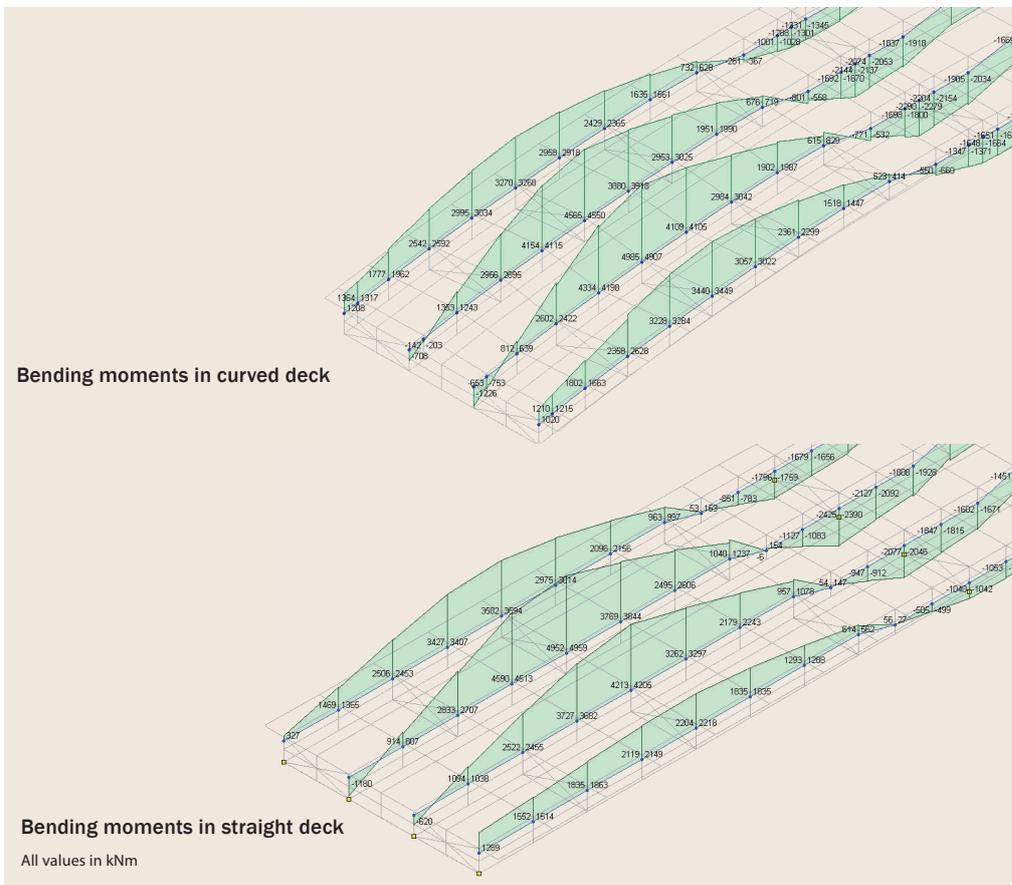


Figure A.6  
Comparison of  
bending moments at  
Stage 4, due to traffic  
loading for maximum  
sagging moment

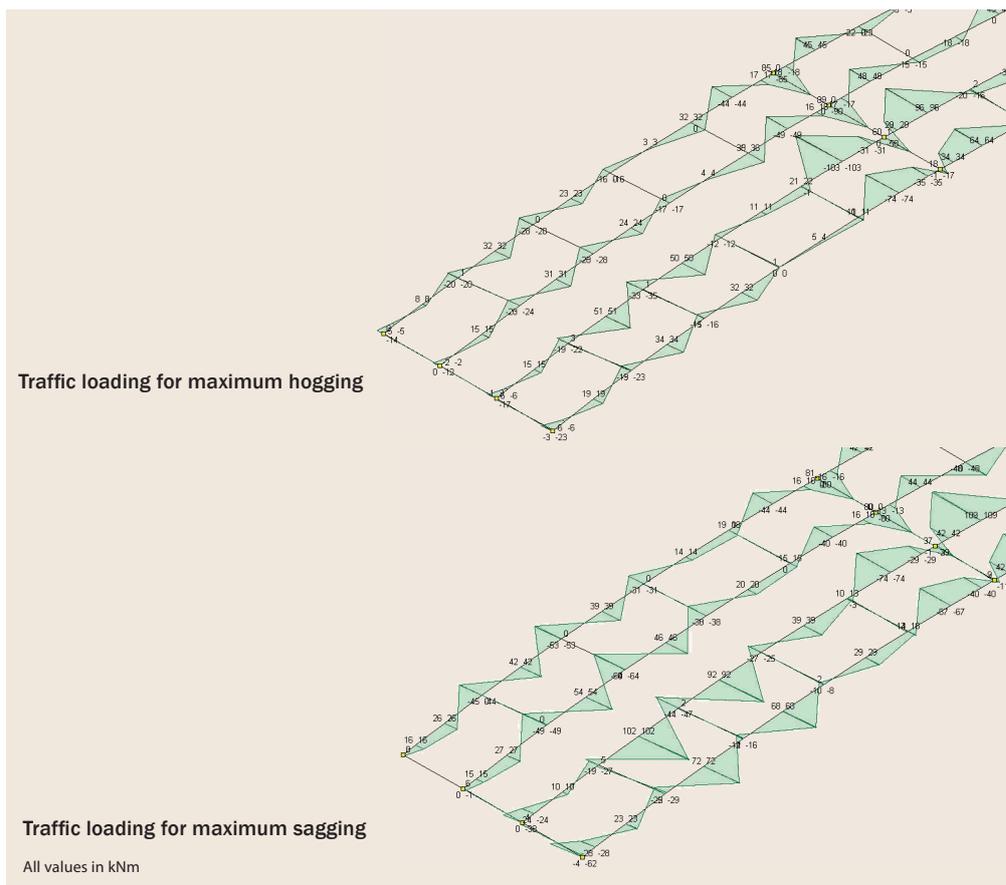


Figure A.7  
Plan bending of bottom flange, due to combined effects, from global model

It is possible to use a finer mesh in the global model, with five divisions between bracing positions but this does lead to a much larger FE model, with implications for the effort needed to create, analyze and interpret the data. For the present example, a sufficiently accurate view of the interaction between effects of major axis bending and lateral bending can be derived, as discussed below.

**Line beam model of bottom flange**

The ‘radial’ component of the force in a curved flange is given simply by dividing the force by the plan radius of curvature. If these components are applied as a linearly varying load to a line beam model, a smoother bending moment diagram results, as shown in Figure A.8 for the maximum hogging combination.

The values local to the intermediate pier are shown at a larger scale in Figure A.9. It can be seen that the variation over the ‘span’ between bracings, i.e. the mid-span value relative to the mean of the values at the two ends, is similar for the global model and the line beam model (120 kNm in this case). In the global model, there will be

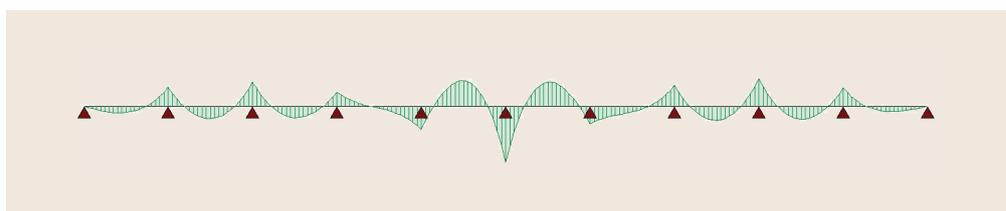
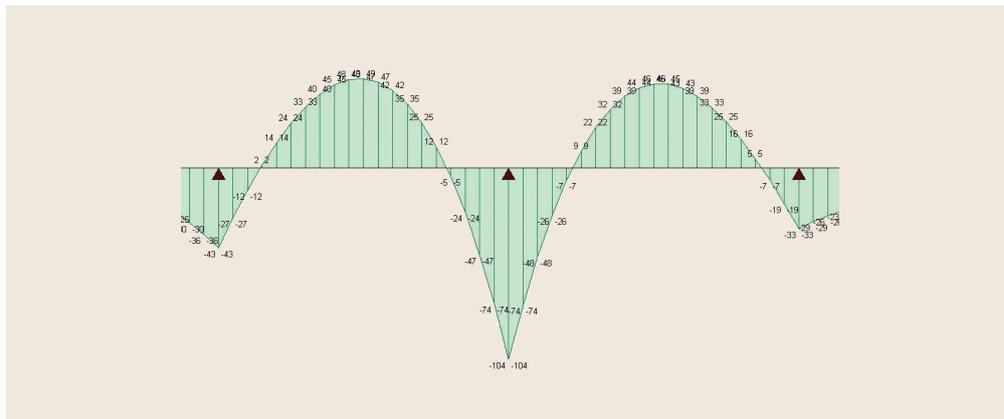


Figure A.8  
Lateral bending of flange, from line beam model

Figure A.9  
Lateral bending moments in bottom flange adjacent to the pier



lateral bending additional effects due to the participation of the bracing with global bending. It is suggested, however, that it would be reasonable simply to consider the peak value of 100 kNm as the maximum value at either the pier or at half way to the first bracing. It may be noted that the value corresponds roughly to  $WL/10$ , where  $W$  is the total 'radial' load over the interval  $L$ .

Applying radial forces corresponding to the loading situation for maximum sagging moment, the corresponding lateral bending moment is about 80 kNm.

### Utilization of lateral bending resistance

The elastic and plastic bending resistances of the bottom flange in transverse bending are 1810 kNm and 1210 kNm respectively at the pier, and thus the utilization ( $M_{z,Ed} / M_{c,Rd}$ ) would be 5.5% and 8.3% respectively. This is modest but significant and needs to be taken into account in the interaction criteria.

In mid-span, the elastic and plastic bending resistances are 565 kNm and 862 kNm, corresponding to a utilization of 14% and 9%.

## A.9 Discussion of deflections during construction

The SLS deflections at stage 1 noted in Table A.5 for the curved deck are illustrated in the deflected profile for Span 1 shown in Figure A.10. The effective transfer of loading toward the girder away from the centre of curvature in each of the pairs results in the outer pair displacing vertically without any significant rotation but the inner pair rotates significantly. This deflection would create significant difficulty during construction, since the inner pair has a differential deflection (between Girder 1 and Girder 2) of 36 mm, equivalent to a slope a 1% ( $0.6^\circ$ ) and the difference between Girder 2 and Girder 3 is 23 mm, or a slope (in the opposite sense) of 0.7%. Without lateral ties between the pairs, there would also be a reduction in width between top flanges of 6 mm.

For this reason, it is likely that control bracing would be provided between the pairs at top flange level and some temporary bracing at bottom flange level, to restrict the differential rotation of the two pairs. Such bracing would affect the vertical bending moments. The effect of this bracing has not been modelled in this example.

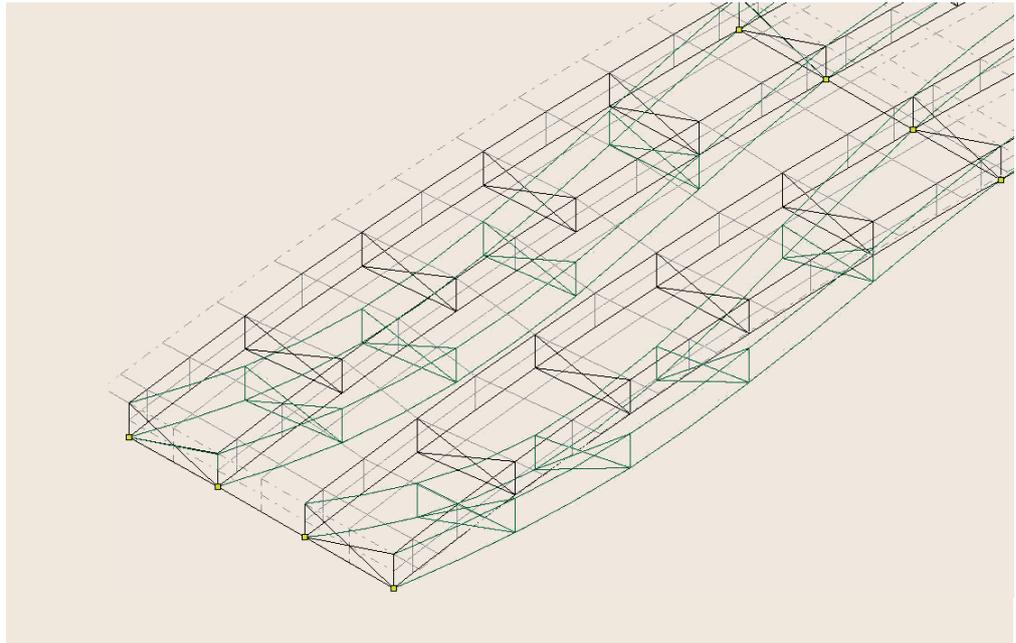


Figure A.10  
Deflected profile at  
construction stage  
with wet concrete  
in Span 1

## A.10 Verification of composite girder

### A.10.1 In hogging bending with axial force

#### *Resistance of the cross section to direct stresses*

The composite cross section is Class 3.

The elastic design bending resistance for a beam constructed in stages depends on the design effects at the stages.

From Table A.7, the design moment on the steel section is 2789 kNm and the total moment is 12196 kNm (only slightly more than in for a straight bridge, in P357), which means that the moment on the composite (cracked) section is 9407 kNm. The stresses are as shown in Figure A.11.

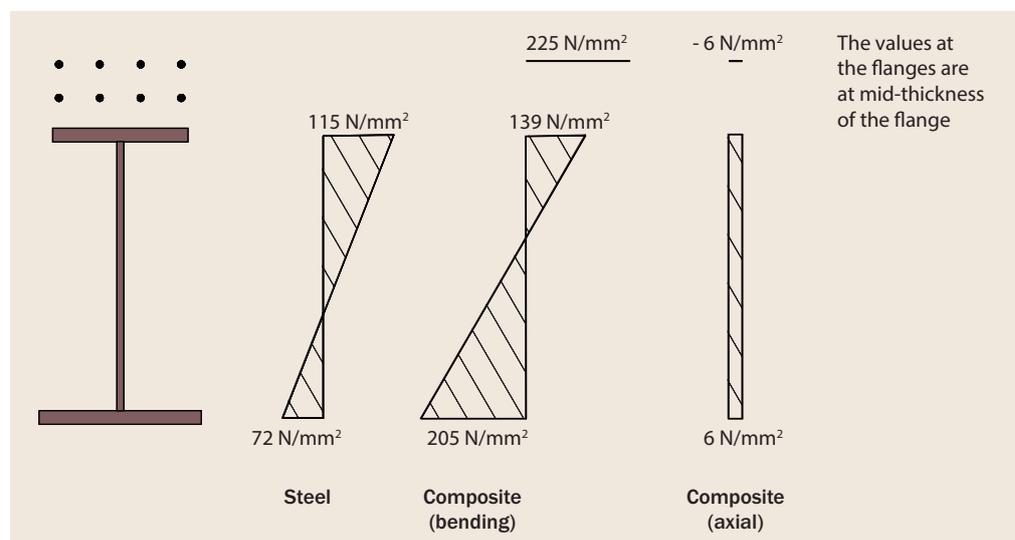


Figure A.11  
Stresses due to  
maximum  
hogging bending

The primary effects of shrinkage do not need to be included.

In addition, the bottom flange is subject to in-plane bending and, as noted in A.8, the maximum moment at the support can be taken as 100 kNm, which results in a bending stress of:

$$f = M_{w,Ed} / W_{el,fl} = 100 \times 10^6 / 3.6 \times 10^6 = 28 \text{ N/mm}^2$$

For verification of cross section resistance, the stresses should not exceed the limiting stresses  $f_{yd}$  and  $f_{sd}$ .

For this verification:

$$f_{yd} = f_y / \gamma_{M0} = 335 / 1.0 = 335 \text{ N/mm}^2 \quad \text{for the 60 mm bottom flange}$$

Since  $28 + 28 = 56 < 335 \text{ N/mm}^2$ , the stress is OK

### **Resistance of the pier-to-first-bracing member to buckling**

For verification of buckling resistance in bending, the design resistance of the cross section (on which  $M_{b,Rd}$  is based) has to be determined using:

$$M_{el,Rd} = M_a + kM_{c,Ed}$$

Where  $k$  is the lowest factor such that a stress limit is reached due to bending alone.

In this case the bottom flange will reach its limit first and the limit is:

$$f_{yd} = f_y / \gamma_{M1} = 335 / 1.1 = 305 \text{ N/mm}^2$$

$$\text{Thus } M_{el,Rd} = 2789 + \frac{(305 - 72)}{205} \times 9407 = 13430 \text{ kNm}$$

To evaluate  $M_{b,Rd}$ , determine the slenderness.

As in P357, the slenderness of the length of beam between the intermediate support and the first bracing into the span can be evaluated using the simplified 'strut model' of EN 1993-2, as allowed by EN 1994-2. As before, the effective Tee section comprises the bottom flange and one third of the depth of the part of the web that is in compression. Assume the same section for the Tee as in P357 and assume that the bracing is at the same location (5.9 m into the span - the distance for the curved example may be slightly different but the difference is small and is neglected here).

Also, the variation of bending moment (and thus axial force in the Tee) is very similar to that in P357 and consequently the following values, taken from P357, may be used:

$$\begin{aligned} N_{crit} &= 97740 \text{ kNm} \\ \bar{\lambda}_{LT} &= 0.363 \\ \chi_{LT} &= 0.877 \\ M_{b,Rd} &= \chi M_{el,Rd} = 0.877 \times 13430 = 11780 \text{ kNm.} \end{aligned}$$

Consider the moment at a distance  $0.25L_k$  from the support, where  $L_k = L/\sqrt{m}$

The total moment is 246 kNm greater than in P357 (an increase of 2.1%), so take  $M_{Ed}$  as 2.1% greater than the previous value, hence at  $0.25L_k$   $M_{Ed} = 10540$  kNm. Take the axial force and resistances as before.

The utilisation for combined vertical bending and axial force is now:

$$\frac{M_{Ed}}{M_{b,Rd}} + \frac{N_{Ed}}{N_{b,Rd}} = \frac{10540}{11780} + \frac{192}{9560} = 0.91 \quad (\text{similar to result for P357})$$

But we must also consider the effect of the additional in-plane bending on the buckling resistance.

As noted in Section 4.1.4, for elastic interaction, the criterion is:

$$\frac{M_{y,Ed}}{M_{y,el,Rd}} + \frac{M_{z,Ed}}{M_{z,el,Rd}} + \frac{M_{w,Ed}}{M_{f,Rd}} \leq 1$$

There is no bending about the minor axis but there is a small axial compression in addition, so the criterion may be restated as:

$$\frac{M_{y,Ed}}{M_{y,el,Rd}} + \frac{M_{w,Ed}}{M_{f,Rd}} + \frac{N_{Ed}}{N_{b,Rd}} \leq 1$$

where  $N_{Ed}$  and  $N_{b,Rd}$  are expressed in terms of values for the effective Tee section.

Thus:

$$\frac{10540}{11780} + \frac{100}{1100} + \frac{192}{9560} = 1.01$$

This is not quite satisfactory and a small increase in flange size would be needed to achieve compliance.

### **Resistance to combined bending and shear**

In P357 it was found that the most critical combination of shear and bending was that for the case of maximum shear with coexisting moment. That loadcase has not been investigated for the curved configuration but the effect of in-plane bending on the limiting envelope is examined below.

From P357, the shear buckling resistance of the web panel (of length 1967 mm) is:

$$V_{bw,Rd} = \frac{\chi_w f_{yw} h_w t}{\sqrt{3} \gamma_{M1}} = 2319 \text{ kN}$$

The web panel length is very similar for the curved bridge. As noted in Section 4.1.2, the shear buckling resistance may be taken conservatively as the lesser of this buckling resistance and the elastic buckling force divided by  $\gamma_{M1}$  which, in this case is equal to 2609 kN (the web slenderness is a little less than 1.0 and for such a value, the resistance is less than the factored buckling force). Consequently it is considered that the buckling resistance does not need to be reduced for plan curvature.

From P357, the resistance moment due to the flanges alone, if coexisting axial force and lateral bending are ignored, is:

$$M_{f,Rd} = 12060 \times 1159 \times 10^{-3} = 13980 \text{ kNm}$$

As noted in Section 4.1.4, the value  $M_{f,Rd}$  in the presence of axial force and lateral bending of the flange is reduced by applying the factor:

$$\left( 1 - \frac{N_{Ed}}{(N_{bf,Rd} + N_{tf,Rd})} - \frac{M_{w,Ed}}{M_{fl,Rd}} \right)$$

Here, the elastic bending resistance of the flange, in-plane is 1210 kNm and the axial resistances of the two flanges are 12060 kN and 17430 kN. The reduced value of  $M_{f,Rd}$  is thus given by:

$$M_{f,Rd} = 13980 \times \left( 1 - \frac{327}{12060 + 17430} - \frac{100}{1210} \right) = 12570 \text{ kNm}$$

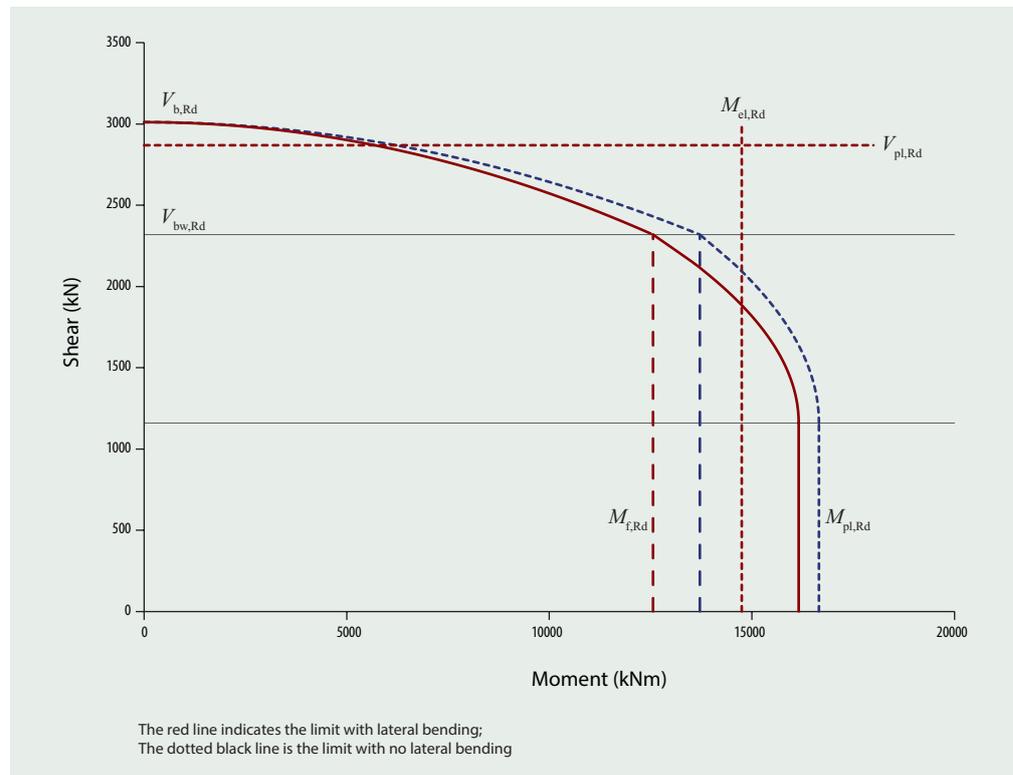


Figure A.12  
M-V interaction limits  
for cross section at  
pier with combined  
bending, axial force  
and lateral bending

Hence, since  $M_{Ed} < M_{f,Rd}$  ( $12196 < 12570$ ), bending resistance does not need to be reduced for shear.

The limiting combinations of  $M$  and  $V$  given by 3-1-5/5.4 and 3-1-5/7.1 are plotted below, for both the curved and straight configurations. The values of maximum moment (in the curved bridge) with coexisting shear is shown on the plot. The value of  $M_{pl,Rd}$  has been adjusted for plastic interaction with the design value of lateral bending moment.

### A.102 Sagging bending

The composite cross section is Class 1 (pna in the top flange) so the plastic resistance can be utilised.

The plastic bending resistance of the short term composite section is 13070 kNm and the total design value of bending effects is 7835 kNm, with a very small axial tensile force. The lateral bending moment is only 80 kNm, compared with a plastic resistance of 862 kNm, so the section is satisfactory by inspection.

In P357 it was shown that the cross section is also satisfactory elastically, taking account of construction in stages. For the curved deck, the vertical bending stresses are slightly greater and there is coexisting lateral bending.

From Section A.6, the design values of stresses are as shown below.

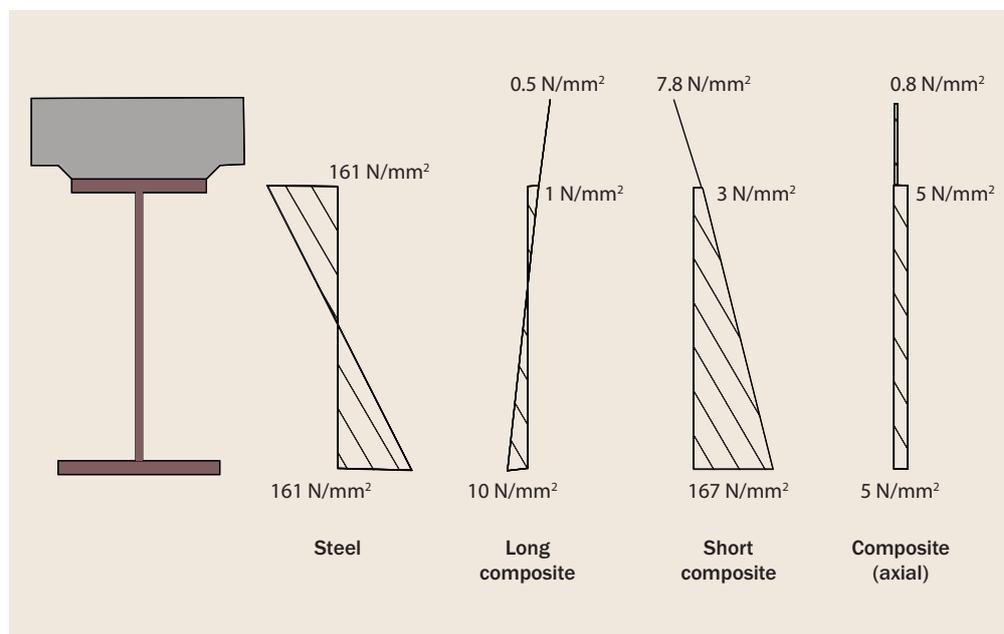


Figure A.13  
Elastic stresses  
at mid-span

The above stresses include the secondary effects of temperature difference (as an accompanying action). The primary effects should be added (values as an accompanying action): they are 4 N/mm<sup>2</sup> compression at the bottom flange and 1.8 N/mm<sup>2</sup> compression at the top of the slab.

For verification of cross section resistance, the stress in the bottom flange should not exceed the limiting stress  $f_{yd}$ .

For this verification:

$$f_{yd} = f_y / \gamma_{M0} = 335 / 1.0 = 335 \text{ N/mm}^2 \quad \text{for the 40 mm bottom flange}$$

The maximum stress is:

$$318 \text{ (vertical bending)} + 5 \text{ (axial)} + 35 \text{ (lateral bending)} = 358 \text{ N/mm}^2$$

So the cross section does not comply with elastic limits (although it is OK plastically, as already noted).









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## DESIGN OF COMPOSITE HIGHWAY BRIDGES CURVED IN PLAN

This publication complements two earlier design guides for the design of composite bridges in accordance with the Eurocodes. It recognizes that many highway bridges carry roads that are on a curved alignment and the supporting structure follows that curved alignment. The guidance addresses the consequences of the plan curvature on the design. The publication discusses:

- The options of using a series of straight girder lengths (chords to a curve) and of using curved girders.
- The behaviour of curved elements, noting the torsional effects that arise, and the application of the Eurocodes to situations that are not always explicitly covered by its rules.
- Consequences for construction (noting that the fabrication of curved girders is readily achievable in modern workshops) and the options for bridge articulation.

A short worked example illustrates the application of design rules to a two-span bridge, similar to the example in publication P357 but curved in plan.

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