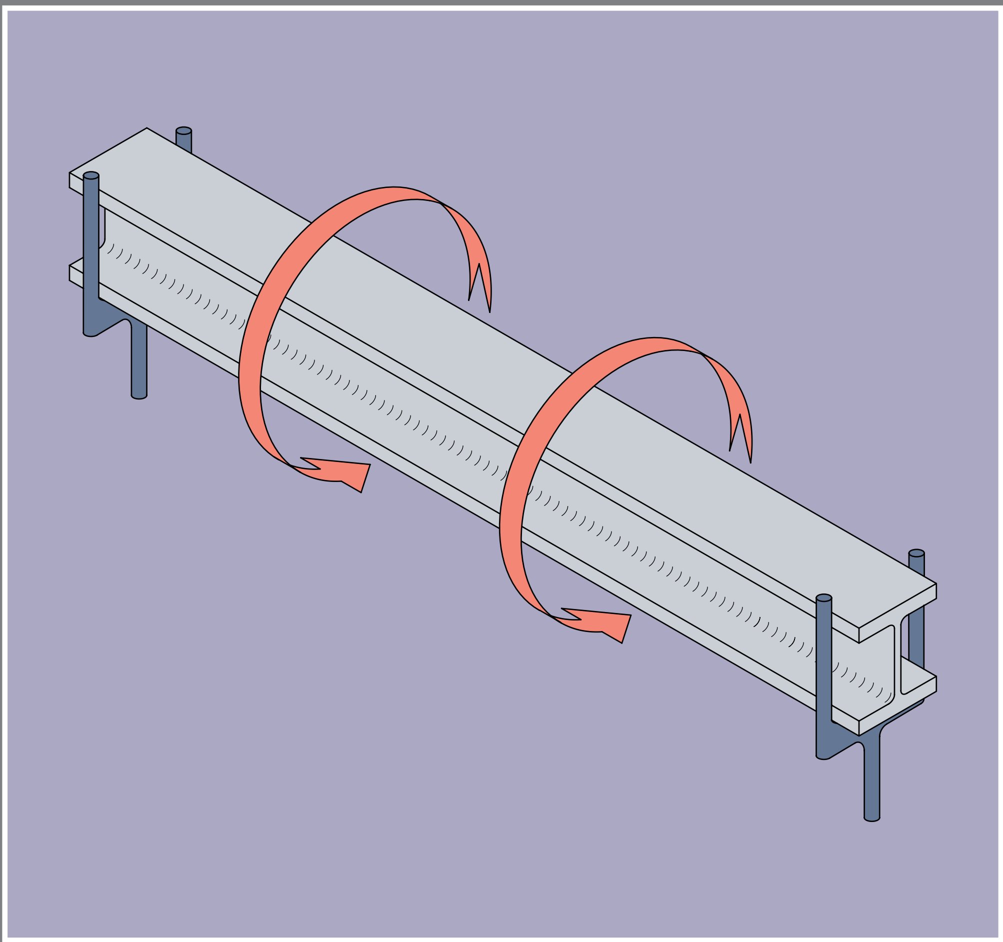
DESIGN OF STEEL BEAMS IN TORSION



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DESIGN OF STEEL BEAMS IN TORSION

DESIGN OF STEEL BEAMS IN TORSION

In accordance with Eurocodes and the UK National Annexes

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FOREWORD

This publication provides guidance on the design of steel beams subject to torsion. It owes much to the earlier SCI publication P057 *Design of members subject to combined bending and torsion* prepared by Nethercot, Salter and Malik and published in 1989. Although the scope is similar and the fundamental theory is unchanged, the guidance has been revised to facilitate design in accordance with Eurocode 3 *Design of steel structures* and to accommodate the changes in the ranges of structural sections for which torsional parameters are provided. The rules for strength verification in Eurocode 3 differ in important respects from those in BS 5950 and there are many changes of terminology and symbolism.

The new publication was prepared by Alastair Hughes, of SCI, with significant contributions from David Iles and Abdul Malik, both of SCI. Account has been taken of feedback from the SCI Members who responded to a request to comment on publication P057.

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CONTENTS

SUMMARY

In most steel-framed structures, beams are subject only to bending and not to torsion but situations do arise where torsional effects are significant, typically where the demands of practical construction result in eccentrically applied loads. The designer will then need to evaluate the magnitudes of the torsional effects and to consider the resistances of the members under the combined bending and torsion.

This publication provides a brief overview of the torsional performance of open and closed structural sections and distinguishes between St Venant torsional effects (sometimes referred to as pure torsion) and warping torsional effects. It explains that the interaction between the two types of effect depends on the torsional parameters for the cross section, the loading and the member length. Expressions and design curves are given for evaluating the two types of effect and guidance is given on the use of simplified approaches that avoid the need for detailed evaluation.

Members subject to torsion will in most cases also be subject to bending. Guidance is given on the verification according to Eurocode 3 of the combined effects due to bending and torsion, both in terms of resistance of the cross section and in terms of resistance against lateral torsional buckling.

Torsional parameters for a range of rolled sections are given in an Appendix. Six short worked examples illustrate the verification for typical design situations.

INTRODUCTION

1.1 Torsion of beams

In most steel-framed structures, beams are subject only to bending and not to torsion. In buildings, beams are usually hot rolled I or H sections, proportioned for optimum bending performance about their major axis. These are 'open' sections and are relatively flexible in torsion; it is usually arranged that the loads on such sections act through the shear centre and thus there are no torsional effects.

However, situations arise where torsional effects are significant, typically where the demands of practical construction result in eccentrically applied loads. For instance, precast units are often supported on one side of a flange or on a shelf angle; in the temporary condition, with one side loaded, most of the load is applied eccentrically. Another example would be a beam which cannot, for architectural reasons, be placed concentrically under the wall it supports.

Faced with such situations, the designer will need to evaluate the magnitudes of the torsional effects and to consider the resistances of the members under the combined bending and torsion. In some circumstances the designer may choose to used 'closed' structural hollow sections, which have a much better performance in torsion; effects and resistances for these will have to be evaluated. At the ends of members subject to torsional loads, torsional restraint must be provided and the connections will have to be designed to resist the forces that provide the restraint.

For simplicity in design and detailing the following approach to steel frame design is suggested:

1. Take all reasonable steps to eliminate torsional effects, avoiding eccentricity by placing beams in line with the loads, or adding beams in another direction to carry the eccentric loads in direct bending.
2. If it is not possible to avoid subjecting a member to significant torsional moment, use a hollow section (typically RHS for a beam), if practical to do so.
3. Where a member is subject to torsion, follow the approach given in this publication to quantify the torsional effects and to verify the member under combined bending and torsion.

1.2 Scope of this publication

Although much of the guidance in this publication is not standard-dependent, it is assumed that the design of steel beams will be carried out in accordance with Eurocode 3, principally in accordance with Eurocode 3 Part 1.1, published in the UK as BS EN 1993-1-1[1] and accompanied by its UK National Annex[2]. A general introduction to design to the Eurocodes is given in SCI publication P361[3].

The elastic theory of torsion has been discussed in many publications and is not repeated here. However, the detailed theory and expressions for determining torsional effects are probably unfamiliar to most building designers. Section 2 therefore sets out a relatively simple summary of the elastic theory of torsion and makes reference to Appendices that provide detailed expressions for evaluating torsional parameters and determining torsional effects in a range of design situations.

Section 3 discusses the design of beams for combined bending and torsional effects, principally in relation to straight I section beams. Particular design considerations for channels and asymmetric beams are given in Sections 4 and 5. A brief overview of the design of structural hollow sections is given in Section 6; the wider considerations for box girders, including distortional effects, are not covered.

Beams curved on plan will be subject to torsion as well as vertical bending. Guidance on the design of curved beams is given in SCI Publication P281[4] and is not discussed within the present publication.

To illustrate the application of the guidance six examples are presented, in calculation sheet format, in Appendix E. These examples illustrate both the simplified approach to determining torsional effects and the detailed evaluation using the expressions in Appendix C.

Section properties for rolled steel sections are given in SCI publication P363[10] but not all the parameters needed for evaluation of stresses due to torsional effects are tabulated there. Appendix A supplements P363 by presenting tables of torsional parameters for UKB, UKC, PFC and ASB sections; the values have been determined using the expressions in Appendix B. Only sections currently produced are included. If properties for older sections are required, reference may be made to the earlier SCI publication *Design of members subject to combined bending and torsion* (P057)[5] or values may be calculated using the general expressions in Appendix B.

Appendix C gives mathematical expressions for determining angle of rotation and its three derivatives for a range of design situations. As explained in the main text, these values are used to determine angle of rotation, St Venant torsional moment, warping torsional moment, and warping moment. For the more common situations, Appendix D presents graphically values derived using those expressions.

1.3 Terminology and symbols

The terminology and use of symbols in this publication generally follows that in the Eurwocodes. Generally, terms and symbols are defined where they are used. Unfortunately, the terms and symbols are not always the same as those used in classical reference texts. The principal terms used in this publication are given below.

Torque is a commonly used term in relation to torsion but here it is used only in the context of an applied twisting moment (an action in Eurocode terms). The symbol T is used.

Torsional moment is the internal twisting moment (about the beam's longitudinal axis). As explained later, it is usually considered in two components, **St Venant torsional moment** and **warping torsional moment**. In Eurocode terms, the design values of the total moment and its two components are symbolized as T_{Ed}, $T_{\mathrm{t,Ed}}$ and $T_{\mathrm{w,Ed}}$ respectively.

Warping Moment is the bending moment in a flange acting as a result of restraint of warping. The moments in the two flanges are equal and of opposite sign. The design value is symbolized as $M_{\mathrm{w,Ed}}$

*Note: The term **Bimoment** is not used in this publication but is found in BS EN 1993-1-1 and is referred to in some texts. It is not a moment but is the product of the warping moment $M_{\mathrm{w,Ed}}$ and the centre-to-centre distance between the flanges. This much misunderstood term, often confused with the warping moment, is not essential to the evaluation of effects and resistances. Where it is mentioned in §6.2.7(4), it effectively means 'due to the restraint of torsional warping'.*

The **angle of rotation** is given the symbol ϕ. Its derivatives $d\phi/dx$, $d^2\phi/dx^2$, $d^3\phi/dx^3$ are symbolized ϕ', ϕ'', ϕ''' respectively.

St Venant torsional constant is the section property relating St Venant torsional moment to the first derivative of rotation (twist per unit length). In Eurocode 3 it is given the symbol I_{T} but in many texts the symbol J is used.

Warping constant is the section property relating warping torsional moment to the third derivative of rotation. It has dimensions of length to the power six. In Eurocode 3 it is given the symbol I_{w} but in many texts the symbol H is used.

Shear modulus. The value of the modulus, G, is given by $G = E/2(1 + v)$, where E is the modulus of elasticity and v is Poisson's ratio. For structural steel, $E/G = 2.6$ and $G \approx 81000 \text{ N/mm}^2$.

Torsional bending constant is given the symbol a and its value is given by $a = \sqrt{EI_{\mathrm{w}}/GI_{\mathrm{T}}}$, where EI_{w} represents the warping stiffness and GI_{T} is the St Venant torsional stiffness. The parameter has dimensions of length. Although this length cannot readily be visualized, it generally expresses the rate at which warping torsional moment diminishes, from a position where warping is restrained. Generally, warping torsional moments are a very small proportion of the total torsional moment beyond a distance of about $3a$ from the position of warping restraint.

1.4 References to Eurocode 3

For brevity, references to BS EN 1993-1-1 are given in the form §6.4.7, which is a reference to clause 6.4.7, and NA.3, which is a reference to clause NA.3 in the UK National Annex. Reference to expressions are given as, for example, Expression (6.21). References to other Parts of Eurocode 3 are given in full.

ELASTIC THEORY OF TORSION

The elastic theory of torsion of uniform bars has been well developed in texts such as Timoshemko[6] and Trahair[7] and the theoretical basis will not be explored here. This Section reviews the elastic theory of torsion from a steelwork designer's perspective, particularly in relation to the torsion of I section beams.

Because all the theory outlined in this Section is elastic, the principle of superposition may be applied when combining effects due to different actions.

2.1 St Venant torsion

A uniform bar or beam that is subject to equal and opposite torques at each end will, if the ends are free to warp out of their planes, resist the torque at each cross section by the pattern of shear stresses shown in Figure 2.1. The total effect of the shear stresses over a cross section is equal to the torsional moment in the beam and the beam will twist about a longitudinal axis known as its shear centre (see Section 2.5 for discussion on the location of the shear centre).

Such behaviour is sometimes referred to as 'pure torsion' but more commonly as St Venant torsion, on account of the theory developed initially by St Venant.

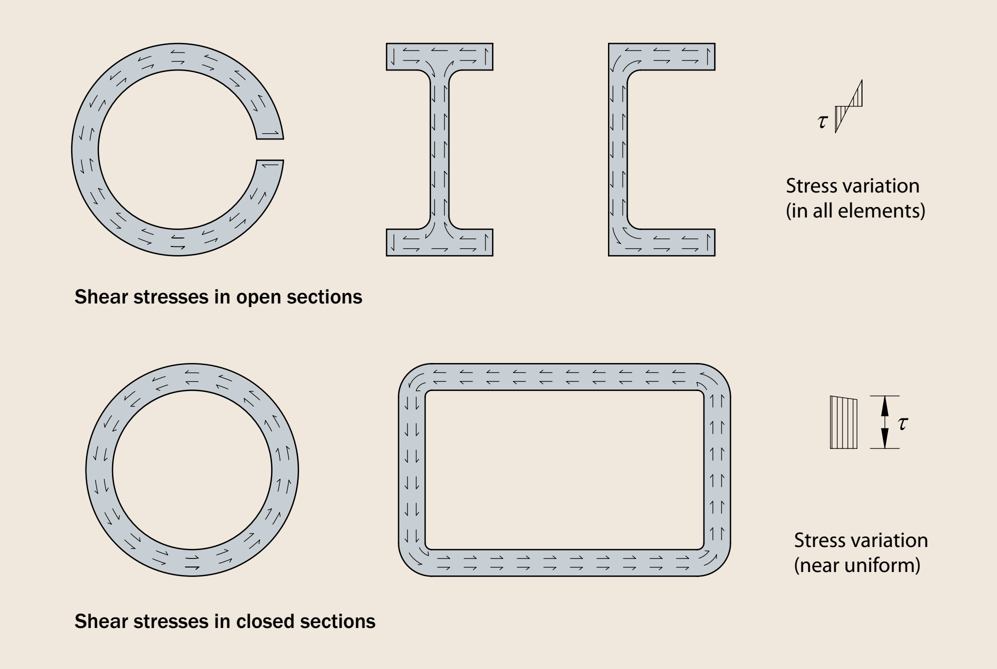


Figure 2.1 St Venant shear stresses

The much greater effectiveness of closed sections in torsion can be appreciated by comparing patterns of shear stresses in open and closed circular sections in Figure 2.1. For the closed section all the shear stresses are in the same rotational direction, thus maximizing their effect. In the open section (the circle with a slit) the shear stresses are in opposite directions at opposite faces and thus are much less efficient in providing torsional resistance.

Cross sections of a circular bar or a circular hollow section will remain plane as a result of uniform twisting but all other sections will experience warping of the cross section, depending on the geometry of the cross section. The warping of solid sections and hollow sections is generally very small and can be neglected. The warping of angle and Tee sections is also very small and can be neglected. The warping of open double-flanged sections, such as an I section or a channel, is much more significant; it is essentially the effect of counter rotation of the flanges in their planes, such as illustrated for an I section in Figure 2.2.

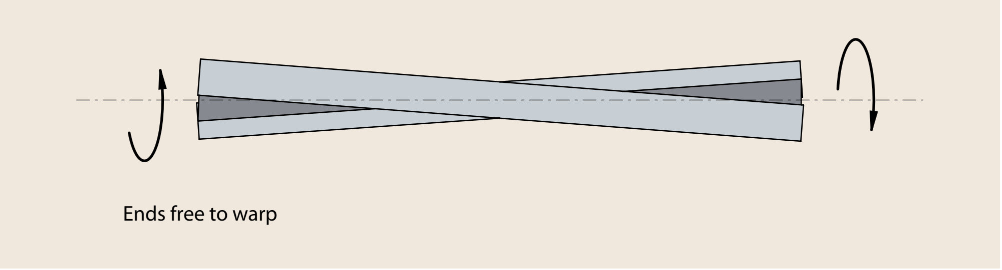


Figure 2.2 Plan view of an I section beam subject to uniform torsion

Twist

The change of rotation (twist) per unit length (i.e. the first derivative of rotation) of a beam due to St Venant torsion is given by:

$$\phi' = T/GI_T$$

where

T is the applied torque

G is the shear modulus

I_T is the St Venant torsional constant.

The rotation ϕ of one end of the bar relative to the other end is thus TL/GI_T.

The above expression for rate of change of rotation is valid for both open and closed sections (but the torsional constant is evaluated differently - see Appendix C for typical expressions for I sections and hollow sections.)

Stresses

St Venant shear stresses are proportional to ϕ'. For an open section, the peak (surface) stress is given by:

$$\tau \qquad = Gt\phi'$$

where t is t_f or t_w as appropriate.

Since $\phi' = T/GI_T$, this can be re-expressed as:

$$\tau \qquad = Tt/I_T$$

or

$$\tau \qquad = T/W_t$$

The parameter W_t is referred to as the torsional section modulus and is similar to the section modulus for bending, except that it gives a value of shear stress rather than direct stress. Its value is not usually tabulated for open sections and the shear stress is simply evaluated as Tt_w/I_T in the web and Tt_f/I_T in the flange.

For a closed section, the same expression ($\tau = T/W_t$) applies, except that the value of W_t is evaluated differently and is generally much greater for a closed section. Values for W_t for structural hollow sections are given in Appendix A and expressions for evaluating W_t are given in Appendix B.4.

Note that the simple expression $\tau = Gt\phi'$ is strictly applicable only to parts of a cross section where the thickness is uniform. If there are sharp re-entrant corners, the St Venant shear stress is increased very locally. This does not require any special consideration for ordinary design at ULS but if the torsion were due to fatigue loading, more detailed assessment should be carried out at such locations. Such advice is outside the scope of this publication.

2.2 Warping torsion

When warping of the cross sections is constrained, longitudinal stresses and additional shear stresses are developed and the torsion is partly resisted by those additional shear stresses. To illustrate the effect of warping restraint, consider a length of uniform I section with a torque applied at the middle. The displacement of the beam would then be as illustrated in Figure 2.3.

If the two halves of the beam had been separate, the left-hand half would have twisted as in Figure 2.2 and the right-hand half would have twisted in the same manner but in

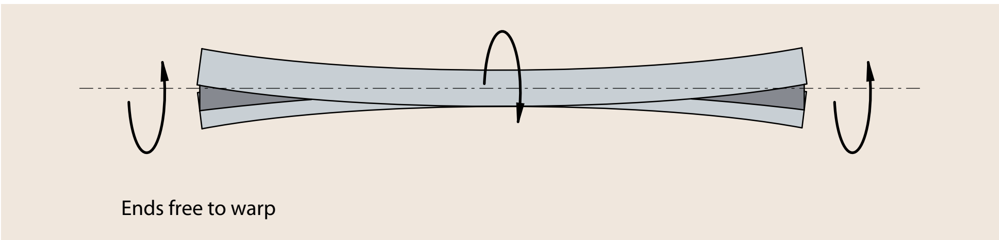


Figure 2.3 Plan view of an I section beam subject to a torque at mid-span

the opposite sense. The warping displacements of the two halves at the middle would be in opposite directions. But because the beam in Figure 2.3 is continuous at mid-span, warping is fully restrained at that location. Both flanges are therefore constrained to bend in plan and the beam will twist at a varying rate over each half span.

At any point in the span, the torsion is carried partly as St Venant torsion (i.e. by the St Venant shear stresses) and partly as warping torsion (i.e. by the shear stresses caused by the restraint of warping). This is expressed in Eurocode terminology (Clause 6.2.7) as:

$$T_{Ed} = T_{t,Ed} + T_{w,Ed}$$

(the suffix Ed denotes design values)

The problem for the designer is how to determine these two design values? The key to this is in formulating a deflected shape that reflects the various stiffnesses.

The separate torsional moments can be expressed in terms of angle of rotation and its derivatives as follows:

$$T_{Ed} = GI_T\phi' - EI_w\phi'''$$

where

T	is the torsional moment at a cross section
ϕ' and ϕ'''	are the first and third derivatives of angle of rotation with respect to distance x along the member
I_w	is the warping constant (for a symmetrical I section $I_w \approx I_z(h - t_f)^2/4$)
I_T	is the St Venant torsional constant.

Formulating the variation of angle of rotation ϕ for the general case where T_{Ed} varies along the beam and allowing for different end conditions is a complex task but for a range of standard situations, algebraic expressions have been derived and these are presented in Appendix C. Some of these are also presented as a series of curves in Appendix D. These curves are readily usable by the designer, without the need to resort to complex calculation.

Warping stresses

Restraint of warping (due either to internal restraint associated with non-uniform moment or to external restraint at the ends) produces longitudinal stresses and shear stresses. For a bi-symmetric I section, warping stresses are shown diagrammatically in Figure 2.4.

The longitudinal warping stresses are greatest at the flange tips and their value is given by:

$$\sigma_w = \pm EW_{n0}\phi''$$

where W_{n0} is the normalized warping function at the flange tip.

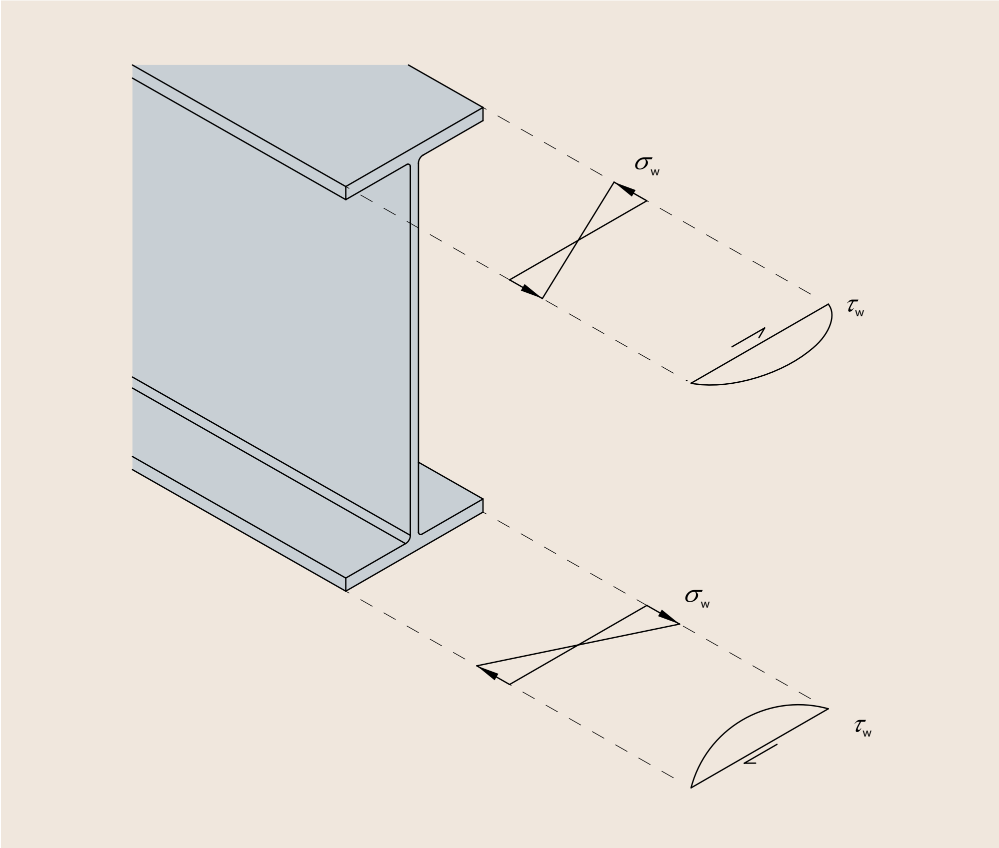


Figure 2.4
Elastic warping stresses in an I section

The warping shear stress is greatest at the junction with the web and its value is given by:

$$\tau_w \qquad = ES_{w1}\phi'''/t$$

where S_{w1} is the warping statical moment.

The terms normalized warping function and warping statical moment, and the symbols used to represent them, have been in use for some time. Although the terms and symbols are not used in Eurocode 3, they are retained here for clarity. Their values depend on the location: for convenience the key locations in the cross section are labelled 0 and 1, for the tips of the flanges and the web/flange junction respectively. (This labelling convention is extended for channel sections - see Section 4.)

Values for W_{n0} and S_{w1} are given in Appendix A.

In practice, for I sections, the warping shear stresses are small enough to be neglected.

For the verification of combined bending and torsion, it is more convenient to use the value of the warping moment in the flange, rather than the longitudinal warping stress. The value of the warping moment is given by:

$$M_w \qquad = EI_w\phi''/(h - t_f)$$

Where $(h - t_f)$ is the distance between the centroids of the two flanges.

For bi-symmetric I sections, this may be re-expressed as:

$$M_w = EI_f\phi''(h - t_f)/2$$

Where I_f is the second moment of area of one flange ($I_f \approx I_z/2$).

Simplified assessment of warping effects

A conservative assessment of warping effects in a flanged beam would be to ignore the St Venant torsional stiffness and to treat the applied torque as a couple of forces F (where $F = T/(h - t_f)$). The warping moment in the flange is then simply calculated as that due to the force F applied to a simply supported beam ($M_{w,Ed} = FL/4$). For long beams, this can be very conservative, as discussed in more detail below.

2.3 Relative magnitudes of St Venant torsion and warping torsion

The above general expression for torsional moment T_{Ed} can be rearranged as:

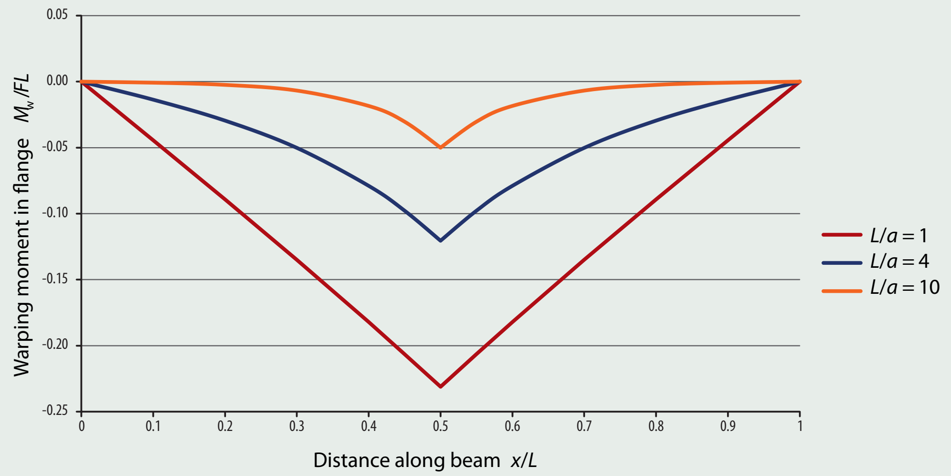
$$T/GI_T = \phi' - a^2\phi'''$$

where $a = \sqrt{EI_w/GI_T}$

The parameter a is known as the torsional bending constant and has the dimensions of length. It is an indicator of how quickly the effect of warping restraint dissipates and may be illustrated by considering the effect in a beam subject to a unit torque at mid-span, as represented in Figure 2.3.

Figure 2.5 shows the variation of St Venant torsional moment for three values of the ratio L/a. In each case the warping torsional moment is the difference between the total torsional moment and the St Venant torsional moment. The curve for $L/a = 1$

Figure 2.5 Variation of St Venant torsional moment in a beam subject to unit torque at mid-span



Ends of beam unrestrained against warping

$F = TI(h - t_f)$

Figure 2.6 Variation of warping moment in a beam subject to unit torque at mid-span

represents a fairly short beam, in which most of the torsion is resisted as warping torsion - i.e. by bending in the flanges. The curve for $L/a = 10$ represents a much longer beam; in which the majority of the beam resists torsion by St Venant torsion.

The magnitude of the warping moment in each flange for these three cases is shown in Figure 2.6. For the short beam, the warping moment is almost equal to that for a simple beam ($FL/4$) but for the long beam it is only 20% of $FL/4$ (for very long beams, M_w tends to $Fa/2$).

From the above discussion, it can be seen that the relative magnitude of St Venant torsional effects and warping torsional effects depend on the torsional bending constant a, which in turn depends on the type of cross section. As a rough guide, Table 2.1 indicates the relative significance of these two means of resisting torsion for a range of section types.

It should also be remembered that the shorter the member, the greater will be the significance of torsional warping (because the L/a ratio is smaller).

Table 2.1 Significance of St Venant torsion and warping torsion for different types of section

SECTION TYPE	SHAPE	ST VENANT	WARPING
Circular hollow sections	○	✓	—
Rectangular and elliptical hollow sections	□ ▯ 0	✓	×
Angles, Tees and cruciform sections	T L +	✓	×
Twin-flanged rolled and fabricated sections	I I [	✓	✓
Thin cold-formed sections	Z Σ	×	✓

Key: ✓ = significant; × = negligible; — = does not act

2.4 Example of the variation of rotation for a cantilever

This example is included to illustrate numerically the variation in rotation and torsional effects along a hot rolled beam cantilever. It might be noted that the behaviour of two such cantilevers, joined back-to-back, would be equivalent to that of a single beam with a central torque, as discussed in general terms in Section 2.3.

Consider the configuration of cantilever of length 1.73 m, using a 305 × 127 UKB42 beam section. For this beam section, the torsional constant $a \approx 1$ m.

The values of ϕ and its derivatives, determined from the expressions in Appendix C are plotted in Figure 2.7 to show how each varies along the length of the member. The following may be noted:

- The plot for ϕ can be viewed as the deflected shape of the flange, in plan.
- The plot for ϕ' shows the variation in twist, to which St Venant shear strains and stresses are proportional, as is the St Venant torsional moment T_t.
- The plot for ϕ'' can be viewed as related to the curvature of the flanges and thus as proportional to the warping moment in one flange. For the other flange, the warping moment is equal and opposite.
- The plot for ϕ''' represents rate of change of curvature and is thus proportional to the warping shear force in a flange. It is thus also proportional to warping torsional moment T_w. Since the sum of T_t and T_w is constant in this example, its shape mirrors that for ϕ'.

If the length of the cantilever were greater, St Venant torsional moment at the tip would be greater; if the cantilever were shorter the St Venant torsional moment at the tip would be less.

2.5 The shear centre

When a member of a steel frame is subject to torsion, this is commonly the result of eccentrically applied load. The torque generated is the product of the force and its perpendicular distance from the shear centre of the section, which is not always its centroid.

For structural sections where there is an axis of symmetry, the shear centre will lie on it. For structural sections which are doubly symmetric, the shear centre and the centroid coincide. Figure 2.8 illustrates the shear centre location for various sections. Dimensions to the shear centre are given in the relevant tables of Appendix A; Appendix B explains how the location is calculated for typical asymmetric shapes.

A member twists about a longitudinal axis through its shear centre. In a beam, important secondary effects depend on the position of the load. Loads applied above the shear centre are 'destabilizing' (because the eccentricity increases as the member twists) and loads applied below the shear centre are 'stabilizing' (see Section 3.3).

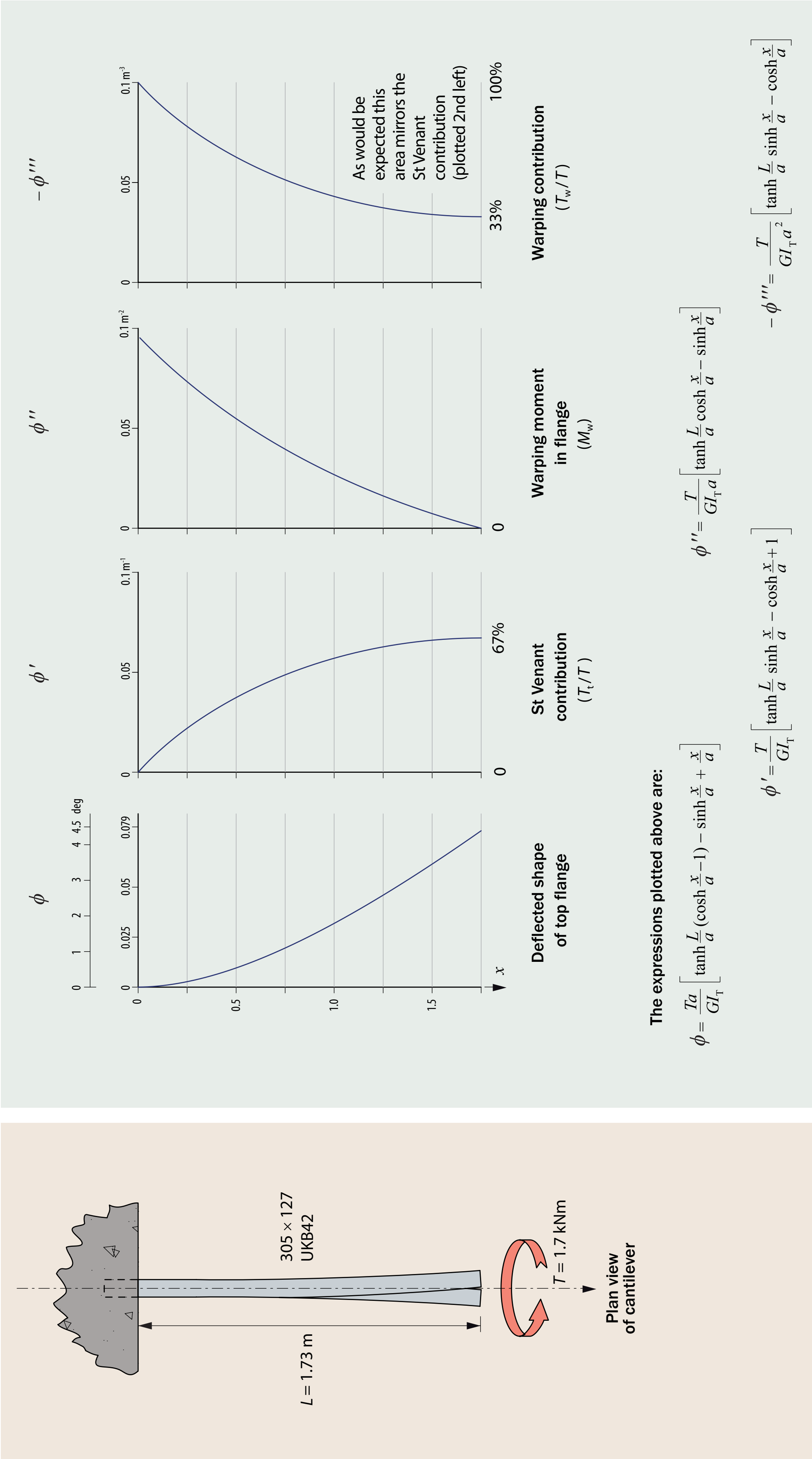


Figure 2.7 Rotation and its derivatives plotted against x, for a selected I section cantilever

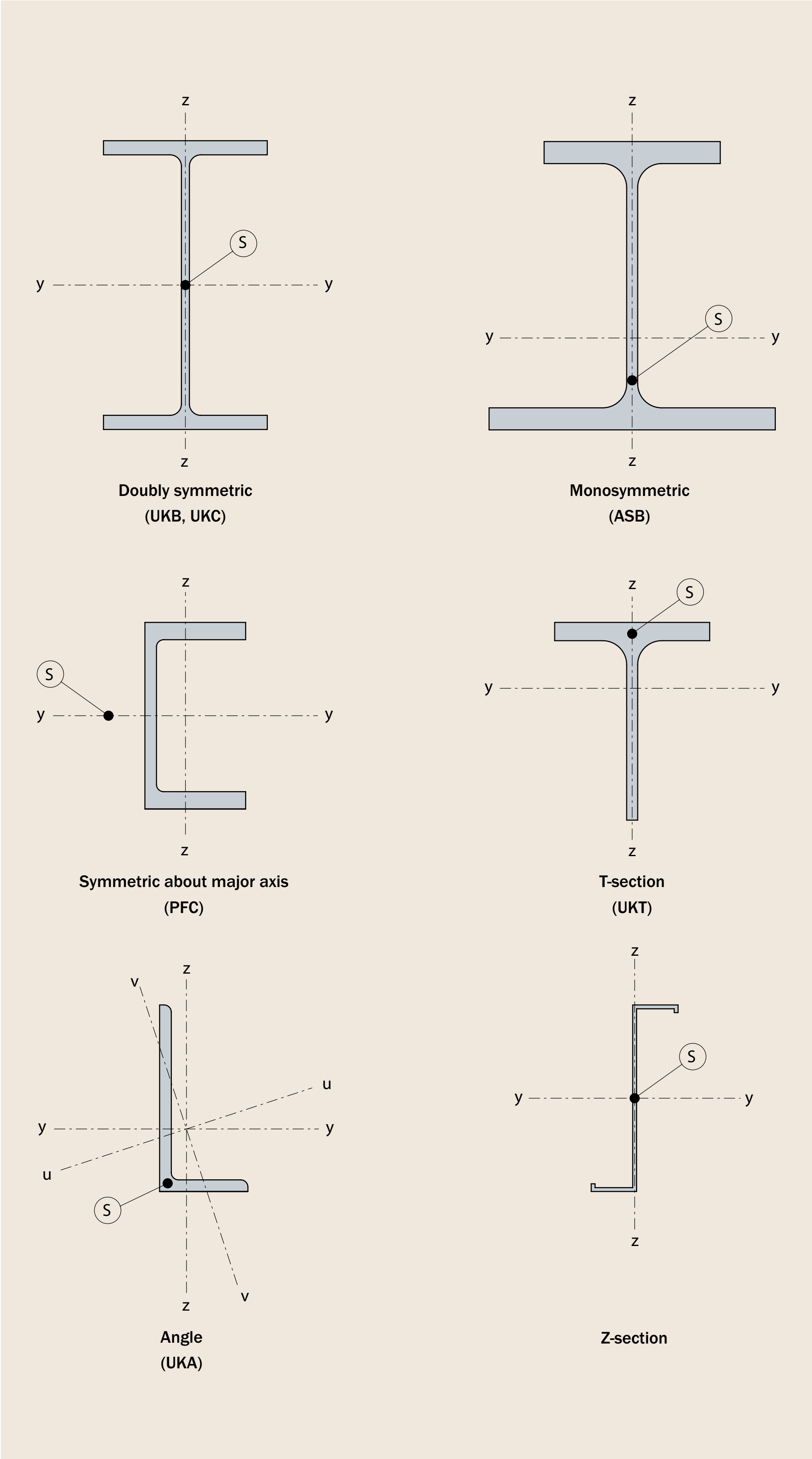


Figure 2.8
Shear centre location for some common structural sections

2.6 Achieving warping restraint at member ends

The warping stiffness of a beam can - in theory at least - be improved by adopting fixed ends. However, normal bolted beam end connections, even those designed to transmit bending moment, cannot be relied upon to provide significant warping fixity. For fixity, it would be necessary for the connection to prevent contra-rotation of the top and bottom flanges in plan, either by clamping them together or by clamping both to another, rigid, element. In practice, this is difficult to arrange.

Connection details designed to provide warping fixity are illustrated in Section 7.6, but these are fabrication-intensive (and therefore expensive) and rarely, if ever, employed in 'normal' building frames. Consequently, no graphs are presented in this publication for warping fixity at the ends of the member, except for the cantilever case (Graph E, Appendix D). Nevertheless, expressions for cases with warping fixity at the ends are given in Appendix C for use in situations where the designer is confident that the needed restraint (to both flanges, or one against the other) can be realized. One case where warping fixity might apply would be a member continuous over two equal spans with identical torsional actions in each span such that warping restraint could reasonably be assumed at the central support by virtue of continuity and symmetry. Continuity alone would not confer warping fixity, as the adjacent span could be loaded in an asymmetric manner.

In reality, flanges attached to full depth end plates do have a certain amount of in-plane rotational restraint from their connection to the adjacent structure. The ends are thus neither restrained nor free but somewhere in between. Elastic theory could generate solutions for less-than-total restraint against warping, even including different restraint conditions for the two flanges (though this would invalidate the hitherto implied assumption that the warping moments, not just the associated shear forces, are equal and opposite). However, the added complication and the near-impossibility of reliable prediction of rotational spring values make the pursuit of 'partial warping restraint' an unattractive one. The choice is between all or nothing, and the safe choice is nothing.

DESIGNING FOR COMBINED EFFECTS

This Section considers the verification of steel beams in accordance with BS EN 1993-1-1, when subject to combined bending and torsion. It is assumed that elastic global analysis is used for determining bending moments and shear forces.

For torsional effects, §6.2.7(3) permits the values of $T_{t,Ed}$ and $T_{w,Ed}$ (St Venant torsional moment and warping torsional moment) to be determined by elastic analysis. Thus the interaction discussed in Section 2 and the graphs and expressions in the Appendices may be used.

Alternatively, §6.2.7(7) allows the simplifications of neglecting torsional warping for a closed hollow section or of neglecting St Venant torsion for an open section. In either case, this completely avoids the process of determining the relative magnitudes of the two types of torsional moment, although this can be conservative for long open section members and is inappropriate for angle and Tee sections (where warping resistance is very small).

At the ultimate limit state, BS EN 1993-1-1 requires verification of the resistance of the cross section and resistance against buckling. For beams, the latter means that lateral torsional buckling resistance needs to be determined; interaction with torsional effects must also be considered.

At the serviceability limit state, BS EN 1993-1-1 and the UK NA only refer to compliance with limits on deflection and vibration. There is no requirement to limit stresses at SLS to the yield strength.

3.1 Resistance of cross sections

According to §6.2.5, the bending resistance of Class 1 and 2 cross sections may be taken as the plastic moment resistance. Class 3 sections can only use the elastic bending resistance. The shear resistance for rolled sections can usually be taken as the plastic shear resistance according to §6.2.6, since such sections are not limited by shear buckling. Bending resistance can be reduced by the presence of a high shear force, according to §6.2.8; biaxial bending is considered in §6.2.9.

Where torsional effects are also present, §6.2.7(1) simply requires that $T_{Ed}/T_{Rd} \leq 1$ but does not give a rule for evaluating T_{Rd}. Additionally, §6.2.7(4) says that the stresses due to torsion should be taken into account, without being specific about how they are to be taken into account. §6.2.7(5) says that for elastic verification, the yield criterion of §6.2.1(5) may be used; but where the section is Class 1 or 2, which most rolled I and H sections are, in bending, the designer will often want to use the plastic bending resistance.

§6.2.7(6) does cover the plastic bending resistance when torsion is present but it only says that the torsional effects B_{Ed} (by which it refers only to the stresses due to warping torsion, not the shear stresses due to St Venant torsion, since B_{Ed} is the bimoment) should be determined by elastic analysis; it does not offer an interaction criterion. In practice, at positions of maximum bending moment the torsional moment is usually wholly warping torsional moment, with no St Venant torsional moment, so the latter does not need to be considered.

3.1.1 Elastic verification

From either the detailed evaluation of the interaction between warping and St Venant torsion or the simplifications allowed in §6.2.7(7), direct and shear stresses can be determined at critical cross sections.

Open sections

Typical stress patterns for a beam loaded eccentrically at mid-span are shown diagrammatically in Figure 3.1 and Figure 3.2.

The maximum direct (longitudinal) stresses occur at the tips of the flanges. At these locations the shear stress is zero (where there is warping restraint, the St Venant shear stresses will generally be negligible and especially so at the tips). The verification according to §6.2.9.2 may be performed. In doing so, minor axis bending due to the rotation of the section (i.e. $M_{\text{z,Ed}} = \phi M_{\text{y,Ed}}$) should be taken into account. The criterion may be expressed in terms of moments:

$$\frac{M_{\text{y,Ed}}}{M_{\text{y,el,Rd}}} + \frac{M_{\text{z,Ed}}}{M_{\text{z,el,Rd}}} + \frac{M_{\text{w,Ed}}}{M_{\text{f,Rd}}} \leq 1$$

Where $M_{\text{f,Rd}} \approx M_{\text{z,el,Rd}}/2$

Shear stresses due to warping torsion are very rarely significant. The (transverse) shear force due to warping restraint is given simply by $T_{\text{w,Ed}}/(h - t_{\text{f}})$. This is usually much less than the (transverse) plastic shear resistance of the flange and may be neglected, as permitted by §6.2.10.

Shear stress due to St Venant torsion will give rise to a small reduction in the plastic shear resistance of the web, according to §6.2.7 (9).

Closed sections

Warping stresses in closed sections are very small and may be neglected. The St Venant shear stresses will also usually be small but where they are significant the interaction permitted by §6.2.10 is not appropriate for this situation, since the

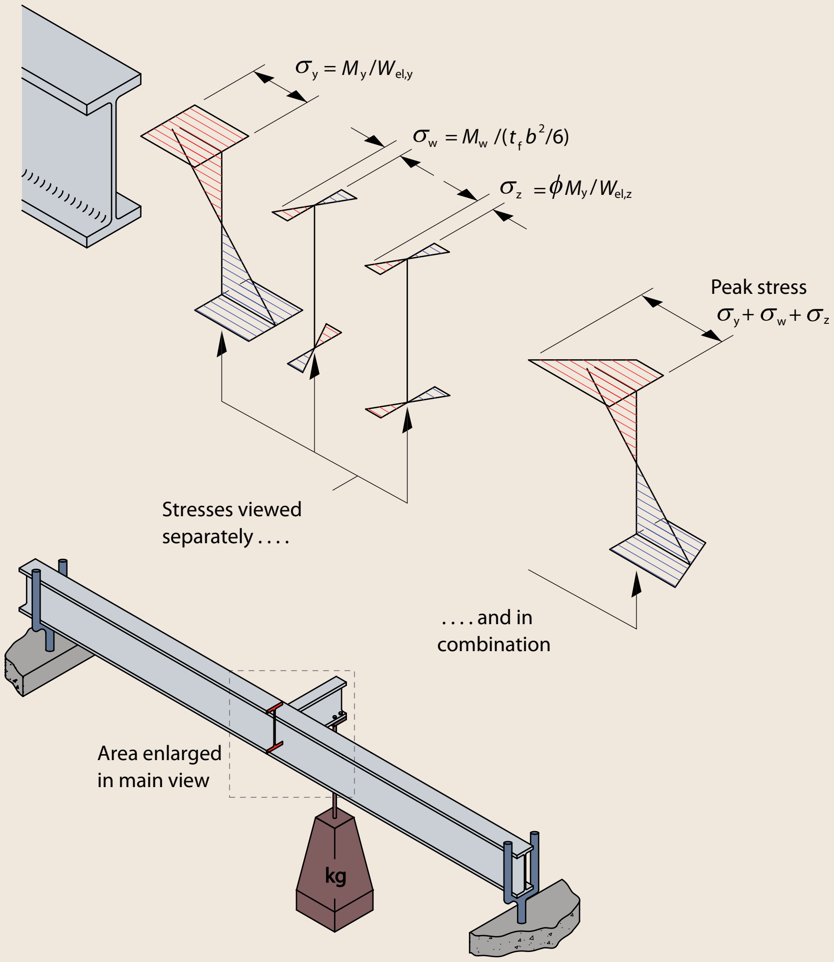


Figure 3.1
Longitudinal stresses in an eccentrically loaded beam shown separately and in combination

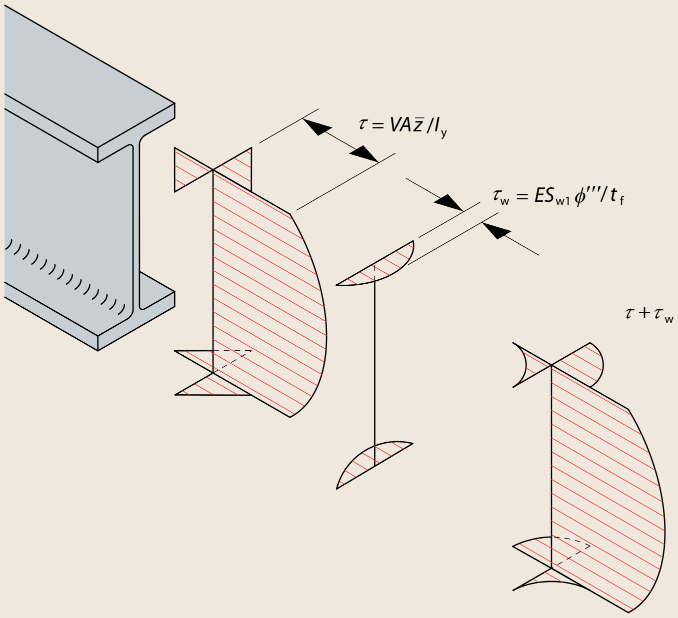


Figure 3.2
Shear stresses due to bending and warping in the beam

shear stress is constant across the flange and will coexist with maximum bending stresses (unlike open sections where peak effects occur at different locations). It is more appropriate to use the criterion in §6.2.1(5), which, in the absence of transverse stresses, reduces to the following:

$$\left(\frac{\sigma_{x,Ed}}{f_y/\gamma_{M0}}\right)^2 + 3\left(\frac{\tau_{Ed}}{f_y/\gamma_{M0}}\right)^2 \leq 1$$

Where $\sigma_{x,Ed}$ is the direct stress (longitudinal) due to biaxial bending.

By inspection, it can be seen that small values of St Venant shear stress ($\tau_{t,Ed}$) would not lead to significant limitation of direct stress.

3.1.2 Plastic verification

Designers will usually wish to utilize the plastic bending resistance of Class 1 and 2 cross sections, for economy. Where there is torsion, direct and shear stresses will usually have been determined elastically. It is therefore necessary to consider the potential effect of the plastification (due to bending) on this determination of torsional moments and on verification of resistance.

Open sections

Where the simplification allowed by §6.2.7(7) for open sections has been adopted, the torsional moment is assumed to be resisted by warping torsion alone; the warping moment in the flange is then easily determined. Minor axis bending due to the rotation of the section (i.e. $M_z = \phi M_{y,Ed}$) must also be taken into account but a note of caution must be given about the value of the rotation of the beam when plastic resistance of the flange is utilized: the rotation will be greater than the elastic value. An allowance for increased rotation should be made, depending on the situation.

Where the interaction between St Venant torsion and warping torsion has been determined according to elastic theory (as in Section 2), it would seem obvious that plastification due to combined major axis bending and warping moment would affect the sharing of the torsional moment. However its effect is to soften the warping stiffness (effectively reducing the value of a) and thus to lead to a reduced value of warping moment. The value of $M_{w,Ed}$ determined by the elastic analysis may thus be used as a conservative value. However, the plastification will also lead to a slightly larger rotation (as noted above) and this should be taken into account when determining the minor axis moment due to rotation.

Where plastic bending resistance is to be utilized, a plastic interaction criterion can be used and the criterion in Expression (6.41) may be adapted for this purpose; assuming that there is no axial force on the beam the criterion is:

$$\left[\frac{M_{y,Ed}}{M_{pl,y,Rd}}\right]^2 + \frac{M_{z,Ed}}{M_{pl,z,Rd}} + \frac{M_{w,Ed}}{M_{pl,f,Rd}} \leq 1$$

where $M_{pl,f,Rd} \approx M_{pl,z,Rd}/2$

Note that this criterion is for a symmetrical I or H section; for channel sections see Section 4 and for asymmetric beams see Section 5.

The (transverse) shear force due to warping restraint is usually much less than the (transverse) plastic shear resistance of the flange and may be neglected, as permitted by §6.2.10.

Shear stress due to St Venant torsion will give rise to a small reduction in the plastic shear resistance of the web, according to §6.2.7(9).

Hollow sections

For hollow sections, the torsion will be resisted as St Venant torsion and the shear stress will be constant around the section, although in most cases the shear stress will be small. The plastic interaction criterion for hollow sections in §6.2.9.1(6) is appropriate for biaxial bending but allowance for the shear stress should be made by reducing the bending resistances using Expression (6.28). This means that the criterion for rectangular hollow sections becomes:

$$\left[\frac{M_{\mathrm{y,Ed}}}{M_{\mathrm{v,y,Rd}}}\right]^{1.66} + \left[\frac{M_{\mathrm{z,Ed}}}{M_{\mathrm{v,z,Rd}}}\right]^{1.66} \leq 1$$

Where $M_{\mathrm{v,y,Rd}}$ and $M_{\mathrm{v,z,Rd}}$ are the bending resistances about the major and minor axes, each reduced by a factor:

$$\left(1 - \frac{\tau_{\mathrm{t,Ed}}}{\left(f_{\mathrm{y}}/\sqrt{3}\right)/\gamma_{\mathrm{M0}}}\right)$$

For circular hollow sections, the bending resistance should be reduced by the same factor.

3.2 Buckling resistance

Where buckling of a member can occur, the buckling resistance must be verified. For steel beams without axial force, lateral torsional buckling (LTB) must be considered, unless the compression flange is continuously restrained. As well as determining buckling resistance for bending about the major axis, interaction with other effects needs to be considered. In BS EN 1993-1-1 Expressions (6.61) and (6.62) provide limiting criteria for the interaction of axial force and biaxial bending; in the absence of axial force these reduce to a simple linear interaction relationship between bending about the two axes.

Interaction of bending with torsion is not covered in BS EN 1993-1-1 but this omission has been addressed in BS EN 1993-6 (concerned with crane supporting structures). In its Annex A it gives a criterion in which the torsional effect and resistance are expressed as the bimoment (see terminology in Section 1.3) but it is perhaps more helpful to re-express the criterion as:

$$\frac{M_{\mathrm{y,Ed}}}{\chi_{\mathrm{LT}} M_{\mathrm{y,Rk}} / \gamma_{\mathrm{M1}}} + \frac{C_{\mathrm{mz}} M_{\mathrm{z,Ed}}}{M_{\mathrm{z,Rk}} / \gamma_{\mathrm{M1}}} + \frac{k_{\mathrm{w}} k_{\mathrm{zw}} k_{\alpha} M_{\mathrm{w,Ed}}}{M_{\mathrm{w,Rk}} / \gamma_{\mathrm{M1}}} \leq 1$$

where:

C_{mz} is the equivalent uniform moment factor for bending about the z-axis according to EN 1993-1-1 Table B.3. (For a simply supported beam with a parabolic bending moment diagram due to uniformly distributed loading, $C_{\mathrm{mz}} = 0.95$; for a triangular bending moment diagram due to a single point load, $C_{\mathrm{mz}} = 0.9$.)

k_{w} $= 0.7 - 0.2\, M_{\mathrm{w,Ed}}/(M_{\mathrm{w,Rk}}/\gamma_{\mathrm{M1}})$

k_{zw} $= 1 - M_{\mathrm{z,Ed}}/M_{\mathrm{z,Rd}}$

k_{α} $= 1/[1 - M_{\mathrm{y,Ed}}/M_{\mathrm{cr}}]$

M_{cr} is the elastic critical moment about the y-axis.

$M_{\mathrm{w,Ed}}$ and $M_{\mathrm{w,Rk}}$ are the warping moment and characteristic bending resistance in the (weaker) flange.

k_{w} can conservatively be taken as 0.7; C_{mz} and k_{zw} can conservatively be taken as 1; but k_{α} does need to be evaluated.

The background to the derivation of this criterion is given by Lindner[8]. While this expression was originally intended for crane runway beams, it may be used for other simply supported beams of uniform cross-section that are subject to torsion.

As noted earlier, bending about the minor axis will result from rotation of the section ($= \phi M_{\mathrm{y,Ed}}$) and this needs to be included in $M_{\mathrm{z,Ed}}$.

3.3 Stabilizing and destabilizing loads

Torsion is more often than not the result of eccentric load. It is preferable to arrange for such a load to be applied at or below the level of the shear centre, the axis about which the member twists, to avoid the secondary effect of increasing eccentricity as the cross-section rotates. If the load is applied below the shear centre (e.g. on the bottom flange) the eccentricity will actually reduce with rotation, though this favourable effect may safely be ignored. However the unfavourable effect of load application above the shear centre (‘destabilizing’ load) must not be ignored.

If necessary, the effect of destabilizing load can be accounted for by repeating the calculation using a magnified eccentricity, determined from the calculated rotation. A single iteration is normally sufficient.

Destabilizing load also affects the elastic critical moment. It can be allowed for by using the freely downloadable software *LTBeam*[9].

3.4 Serviceability limit state

3.4.1 Limiting criteria for rotation

EN 1990 sets out the principle that 'serviceability criteria for deformations and vibrations shall be defined' but only mentions vertical and horizontal deflections in general terms; no mention is made of rotation. BS EN 1993-1-1 offers no recommendations for deflection limits of beams and the UK NA only offers suggested limits for vertical and horizontal deflections; again, no mention is made of rotation.

In P057, it was suggested in a footnote to one of the worked examples that a 2 degree limit to the angle of rotation would seem appropriate. The intention was to offer practical advice without being definitive. This 'limit' has been in print for over 20 years and SCI's Advisory Desk has directed enquirers to it. There has been little feedback on its application in practice, successful or otherwise, but, in the absence of any other guidance, it may be accorded some respect by virtue of long existence without negative comment.

A note of caution is needed, perhaps, where facades are concerned. A rotation of 2 degrees under a 4 m high masonry wall translates into a 14 mm displacement at the top, which seems unacceptable. It would be hard to resist the conclusion that a more restrictive limit should apply in sensitive situations. What that limit is must continue to be a matter for case-by-case judgement.

3.4.2 The likelihood of serviceability governing

An I section is generally very flexible in torsion and the limitation of rotation at SLS is likely to govern when there are significant torsional moments. Hollow sections are very stiff in torsion but if the torsional moments become a significant proportion of the torsional resistance, the rotation would nevertheless be large – if, for example, a square hollow section were designed to use the full torsional resistance at ULS, then at SLS a twist of 2° would be generated over a relatively short length of about 10 times its width.

It should also be noted that a 2° rotation at SLS would be about 3° at ULS and that rotation would introduce a minor axis moment of about 5% of the major axis moment.

DESIGN OF CHANNELS

Channels are open sections and, like I sections, are flexible in torsion and resist it by a combination of St Venant and warping torsion. However, because of their asymmetry about the z-axis, they are more likely to be subject to torsion than are I sections.

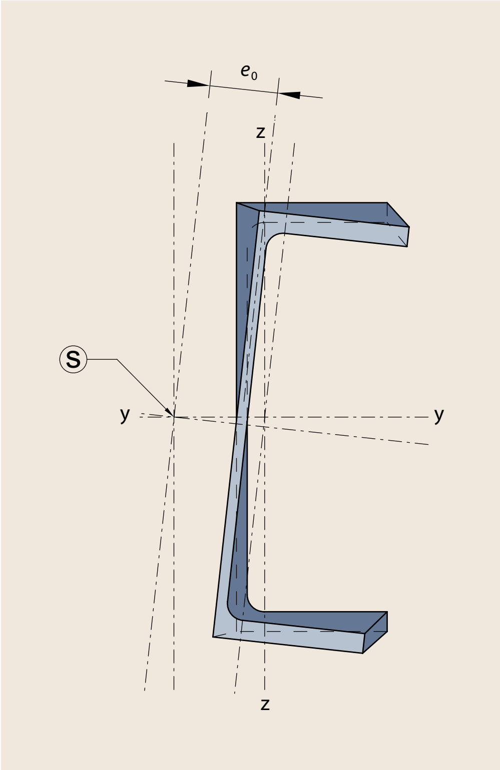


Figure 4.1
A channel twisting about its shear centre

A channel will twist about its shear centre, which lies outside the section on the web side (as shown in Figure 4.1). A channel loaded on its top flange or directly over its web would, according to elastic theory, be subject to torsion. Only if load acts in line with the shear centre would it be torsion-free. Dimensions from the centroid to the shear centre are given in Appendix A for parallel flange channel sections (UKPFC).

When a channel section is subject to bending due to a point load that acts through the shear centre and parallel to the web, the bending stress in the flanges is uniform across their width and the shear stress varies as shown in Figure 4.2.

4.1 St Venant torsion

As for other open sections, the twist per unit length due to uniform torsion when warping is not restrained (i.e. St Venant torsion) is given by $\phi' = T/GI_T$ and the St Venant shear stress is given by $\tau = Tt/I_T$. Values of I_T for UKPFC sections are given in Appendix A.

4.2 Warping torsion

Channels possess warping resistance in the same way as do I sections, though they have narrower flanges. The web participates in the warping resistance, effectively forming an L section with each flange. The variation of warping stresses in a channel section is shown in Figure 4.3. As for I sections the magnitudes of the direct and shear

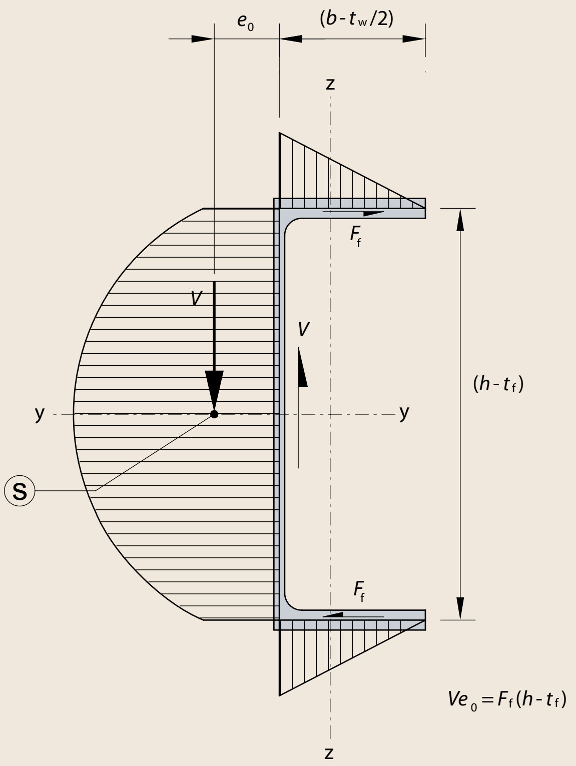


Figure 4.2 Shear stress distribution for a channel loaded through its shear centre

stresses depend on the normalized warping function and warping statical moment parameters but there are additional key locations for these parameters, as shown in the Figure. The peak warping stress occurs at the flange tip (where $\sigma_w = EW_{n0}\phi''$) and the peak shear stresses occur at location 1 in the flange ($\tau_w = ES_{w1}\phi'''/t_w$) and at the top of the web ($\tau_w = ES_{w2}\phi'''/t_w$). These effects are summarized in Table 4.1.

Where plastic resistance is considered when evaluating the interaction of bending with torsion (see Section 3.1.2), the plastic resistance to warping moment should be based on the flanges alone, ignoring the web.

4.3 Practical considerations

Despite the eccentric location of the shear centre, there is little evidence of channel section beams rotating when they are directly loaded by concrete slabs on the top flange. This is because the top flange is restrained against horizontal movement by friction and the bottom flange is under tension. For short span trimmer beams it is common practice to ignore torsional effects and to rely on this restraint.

UKPFC sections are sometimes used as lintels, placed under the inner leaf of a cavity wall with a plate welded to the bottom flange (or an angle welded to the web) to provide unobtrusive support for the outer leaf. Customarily, the effect on the shear centre of the additional bottom flange is ignored. Example 4 (Appendix E) demonstrates this approach.

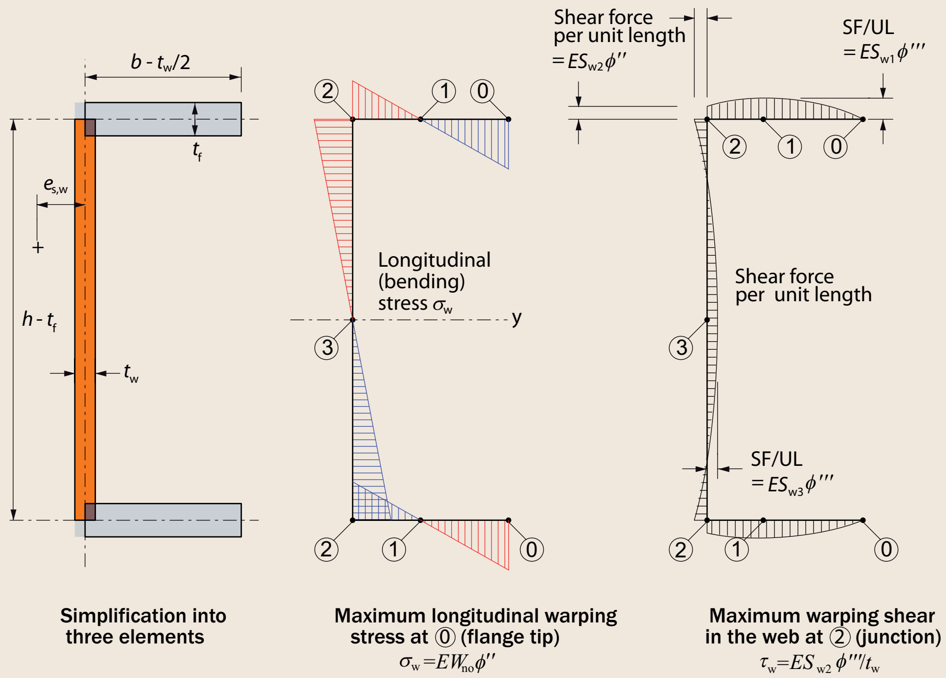


Figure 4.3 Elastic warping effects in a channel section

Table 4.1 Peak elastic stresses due to torsion in a channel section

	STRESS	IN WEB	IN FLANGE
ST VENANT	Longitudinal	NONE	NONE
	Shear (at element surface)	$\tau_t = Gt_w\phi'$	$\tau_t = Gt_f\phi'$
WARPING	Longitudinal	$\sigma_w = EW_{n2}\phi''$	$\sigma_w = EW_{n0}\phi''$
	Shear	$\tau_w = ES_{w2}\phi'''/t_w$	$\tau_w = ES_{w1}\phi'''/t_f$

For the location of points 0, 1 and 2 see Figure 4.3

DESIGN OF ASYMMETRIC BEAMS

Beams used in *Slimflor*® and *Slimdek*® solutions are subject to torsion at various stages of the construction sequence. With both fabricated and rolled asymmetric sections, the shear centre is below mid-depth and the calculation of warping moment is slightly more complicated than for doubly symmetric I sections.

5.1 Types of asymmetric beam

Where a suitable ASB rolled section is available, it will usually provide the most cost-effective solution. For details of the ASB range, see SCI publication P363[10] or the Tata Steel website.

Slimflor® fabricated beams (SFB), comprising a 15 mm plate welded to the underside of a UKC section, offer a wider range of asymmetric beam sizes, providing bottom flange outstands of 100 mm for the deep decking or precast unit.

A third option is to fabricate a bespoke section from three plates. This allows near-total freedom of dimensional choice, constrained only by available plate thicknesses.

5.2 Section properties

Section properties for ASB sections are given in P363, in Tata Steel literature and the Tata Steel website. However, not all the necessary torsional parameters are given in those sources. Torsional parameters are therefore given in Appendix A. It will be noticed that some of the values given in Appendix A differ slightly from values given in the other sources. The values in this publication have been accurately calculated using Appendix B and should be used in preference to those in the other sources, where they differ.

Property tables for a range of 36 *Slimflor*® fabricated beams (SFB) are published by Tata Steel[11].

For sections welded from three plates, I_T may be conservatively approximated as $\Sigma(Lt^3/3)$ for the three constituent rectangles. A formula for I_w is given in Appendix B.3.

5.3 Transient and permanent design situations

Asymmetric beams employed in *Slimflor*® solutions may be used together with either deep decking or precast units, as illustrated in Figure 5.1. The beams can be subject to considerable torsional loading at various stages, such as when the wet concrete and constructional loads are present on one side only.

Internal beams are normally designed for this condition (as a transient situation) and are verified for lateral-torsional buckling resistance; it is not then necessary to restrict the construction sequence to one that maintains balanced loading. It is generally considered impractical and potentially unsafe to do otherwise. On the other hand, where precast units are used it would normally be considered both practical and safe to require that the units are in place on both sides before in-situ topping is cast (which might be on one side before the other). Design Example 3 in Appendix E illustrates a range of temporary load cases.

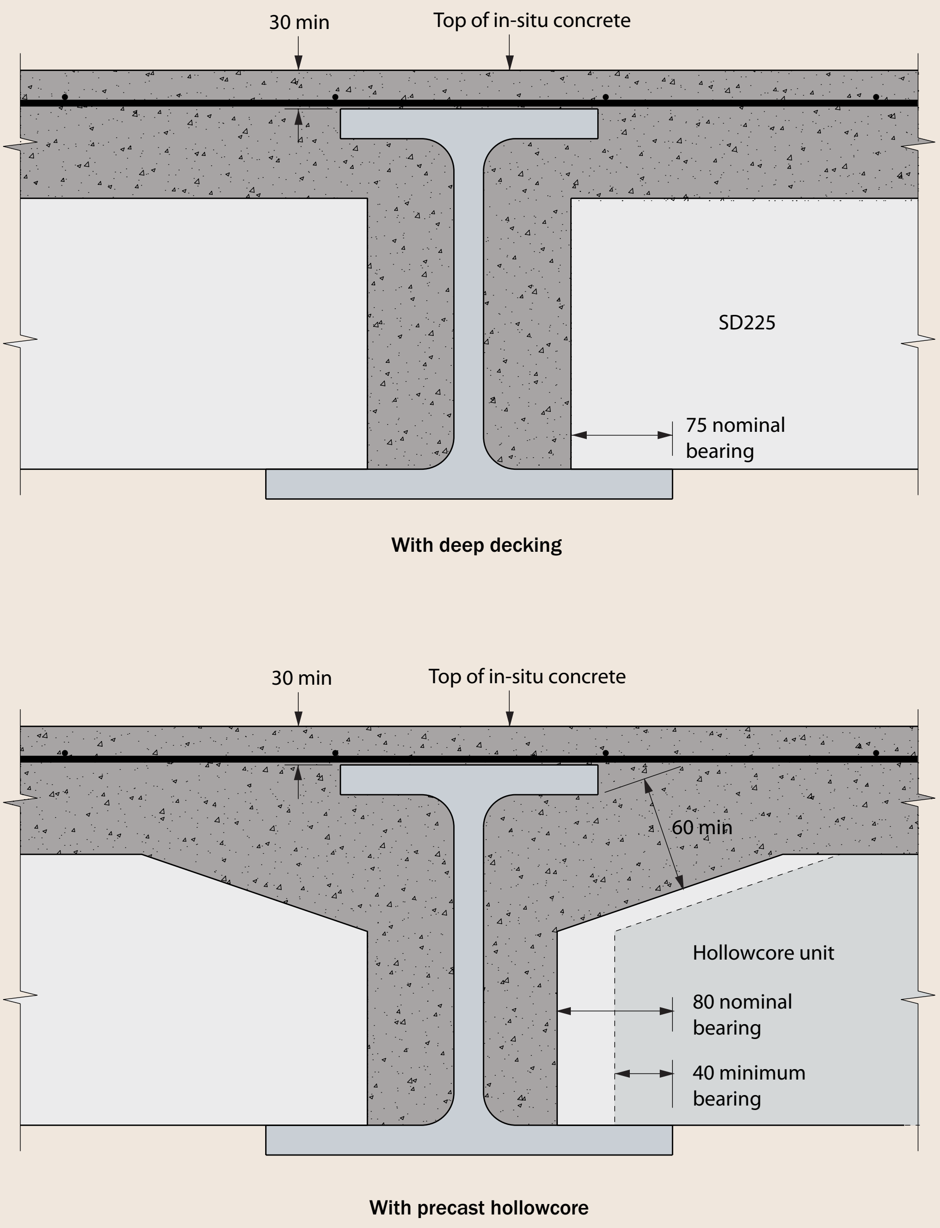


Figure 5.1 Typical solutions using ASB sections (all dimensions in mm)

Once the in-situ concrete has hardened, an internal beam may be considered to be restrained against lateral-torsional buckling. Further, it would seem reasonable, where the section is fully surrounded by infill concrete on both sides, to rely on this floor plate for restraint against twisting under load imbalance. Accordingly, in the final condition, a typical internal beam need only be verified for resistance of its cross section in bending and shear.

Edge beams will need to be verified for the effects of torsion in the final condition unless they are effectively tied to the adjacent slab at low level (which could be a cost-effective alternative, especially if the ties serve more than one function). Beams with deep decking spanning parallel on one side should be treated as edge beams in this respect, whereas parallel precast units can be considered to provide restraint.

For an edge (or quasi-edge such as when parallel decking is present) beam in the final condition, a top flange set against a plate of hardened concrete is obviously not going to deflect inwards. Effectively, the shear centre is forced up to top flange level. Any rotation must be due to the bottom flange deflecting outwards, having overcome considerable (but unreliable) bond and friction resistance. In warping terms, the full flexural rigidity and resistance of the larger flange can act at a lever arm of $(h - t_f)$ to oppose torsion. It is clear that results from a conventional torsion calculation will be very conservative.

5.4 Design effects

The shear centre of a monosymmetric beam remains on the z-axis (the axis of symmetry) but is offset from the centroid, towards the bottom flange but still above it. This means that load applied at bottom flange level is not destabilising (though the beam’s own weight is).

Beams used in *Slimflor®* solutions are normally not subject to the rotation-induced weak-direction bending which would apply to a beam with a freely suspended eccentric gravity load. This is because, for the case where the beam supports the decking or precast unit (i.e. the decking or unit is transverse to the beam) the decking (if it is adequately fixed to the beam) or the precast unit provides the lateral restraint necessary to resist this component.

Asymmetry does not change the process of calculating ϕ and its derivatives, which is carried out as for a doubly symmetric section using tabulated values of I_t and a. The difference arises at the stage of quantifying the warping moment in the flanges. The following expression should be used for the warping moment in the top flange (the value of $M_{w,Ed}$ is numerically the same for both flanges but it is the top flange that governs).

$$M_{w,Ed} = EI_{tf}\phi'' e_{s,f}$$

where $e_{s,f}$ is the height of the centroid of the top flange above the shear centre and I_{tf} is the second moment of area of the top flange (bending in its plane).

5.5 Verification at ULS

Verification of the cross-section resistance follows the same procedure as for a symmetric I section beam. ASB sections are Class 1 sections and the plastic interaction criterion given in Section 3.1.2 may be used.

Buckling resistance will nearly always need to be considered with a *Slimflor*® beam at the wet concrete stage, since anything which could provide restraint (such as a precast slab unit) is attached below the shear centre. The interaction formula from BS EN 1993-6 (discussed in Section 3.2) is valid for asymmetric sections. *LTBeam*[9] can compute M_{cr} for asymmetric I sections (as explained in an article[12] giving guidance for first-time users). *LTBeam* will allow for the favourable (stabilizing) effect of loads applied below the shear centre, and provides for a second UDL at a higher level, to represent the beam's self weight. Moreover, it computes the levels of the centroid and the shear centre for this purpose.

For ASB sections, the requirements in NA.2.17 mean that curve a in Table 6.4 should be used to determine the reduction factor χ_{LT}. For a beam welded from three plates, §6.3.2.2 leads to the use of either curve c or d, depending on the h/b ratio.

5.6 Serviceability limit state

Previous design guidance for *Slimflor*® solutions[13], [14] has suggested limiting lateral deflection of the top flange to span/500. For a typical 7.5 m span this is 15 mm, which could correspond to over 3 degrees rotation. While this is higher than might be judged tolerable in other situations, no problems have been reported in practice. This is the limit applied in the temporary (wet concrete) load case. Since part of the loading (the construction loads) disappears before the concrete sets, the twist actually locked into the completed structure is due to the weight of concrete alone.

In the case of an edge beam in its final condition, the top flange is prevented from deflecting inwards by hardened concrete. Even if torsion were significant in the verification at ULS, the prime concern for SLS is likely to be vertical deflection. Note that some outwards deflection of the bottom flange may have been locked in due to the eccentric loading during construction but with typical asymmetric beam proportions any intrusion into the cladding zone should not exceed 20 to 30% of the span/500 limit recommended for the top flange.

STRUCTURAL HOLLOW SECTIONS

With structural hollow sections, torsional warping displacements are generally negligible or absent (in CHS) and torsional moments are entirely St Venant effects. Distortion of the cross section of rectangular hollow sections might occur when eccentric moments are introduced through a connection on only one face of the section but such effects are outside the scope of this publication. Torsion of box girders is also outside the scope of this publication.

6.1 Elastic behaviour of hollow sections

As can be seen from the Tables in Appendix A and the expressions for torsional constant I_T in Appendix B, the torsional stiffness of hollow sections is much greater than that of open sections of comparable size. The same general expression for torsional shear stress applies, that is $\tau = T/W_t$, but now the value of W_t is much greater than for an open section. Strictly, the torsional shear stress is slightly greater on the outer surface than on the inner but the difference is small in a thin-walled section and in practice the stresses would redistribute at yield.

6.2 Resistance to combined bending and torsion

Since structural hollow sections are often chosen to resist large torsional moments, the torsional resistance may be substantially utilized in such cases and this would affect the resistance to bending and shear.

In §6.2.7(9) the plastic shear resistance is reduced due to the presence of St Venant torsional shear stress. The reduction factor for hollow sections makes a simple reduction that is appropriate when the vertical shear and torsional shear act in the same direction; there is no benefit from the fact that the shears act in opposite directions on the other side of the hollow section.

There is no mention in BS EN 1993-1-1 of how to take account of the effect of large St Venant shear stresses in the flanges of hollow sections on the bending resistance but, as noted in Section 3.1.2, it would seem appropriate to apply to the flange area the same factor as applied in Expression (6.28) to $V_{pl,Rd}$.

For almost all practical situations lateral torsional buckling is not a concern for RHS sections. According to ECCS Publication 119[15], RHS sections may be considered to be non-susceptible up to a non-dimensional slenderness of $\bar{\lambda}_z = 10b/h$. Even with $h/b = 3.333$, currently the slimmest RHS in the range produced by Tata Steel, this corresponds to an uncommonly high slenderness.

It should also be noted that a few RHS sections have wall thicknesses that would make them susceptible to shear buckling. According to Clause 5.1(2) of BS EN 1993-1-5[16], a plate is susceptible to shear buckling if the value of h_w/t exceeds $72\varepsilon/\eta$ (= 59 for S355 steel, with $\eta = 1$, as set by the UK National Annex). For the current (2010) Celsius® SHS range, only 400 × 150 × 6.3, 400 × 200 × 6.3, 500 × 200 × 8 and 500 × 300 × 8 sections exceed this limit. Any reduction for shear buckling should be applied to both the vertical shear resistance and the resistance of the RHS to St Venant shear.

DESIGN OF CONNECTIONS

End plates offer a simple solution where a bolted connection is required to transmit torque. An arrangement of four or more bolts will be able to resist a combination of torque and shear force. End plate connections with relatively thin end plates may be considered as nominally pinned connections in simple construction. Where thicker end plates are used in moment-resisting connections, the bolt tension forces will reduce their shear resistance.

As an alternative to an end plate connection, cleats to both flanges would provide reliable torsion resistance. Other connection types traditionally associated with simple construction (fin plates or double angle cleats) are best avoided for connections designed to transmit more than nominal torsion.

7.1 Types of end plate connection

There are three basic types of end plate connection: partial depth, full depth and extended. Partial depth end plates are not suitable for providing significant torsional resistance, because they are not connected to both flanges. Typical full depth and extended end plate connections are shown in Figure 7.1.

If the beam is a hollow section, the end plate can extend horizontally or vertically, depending on what is available to connect to.

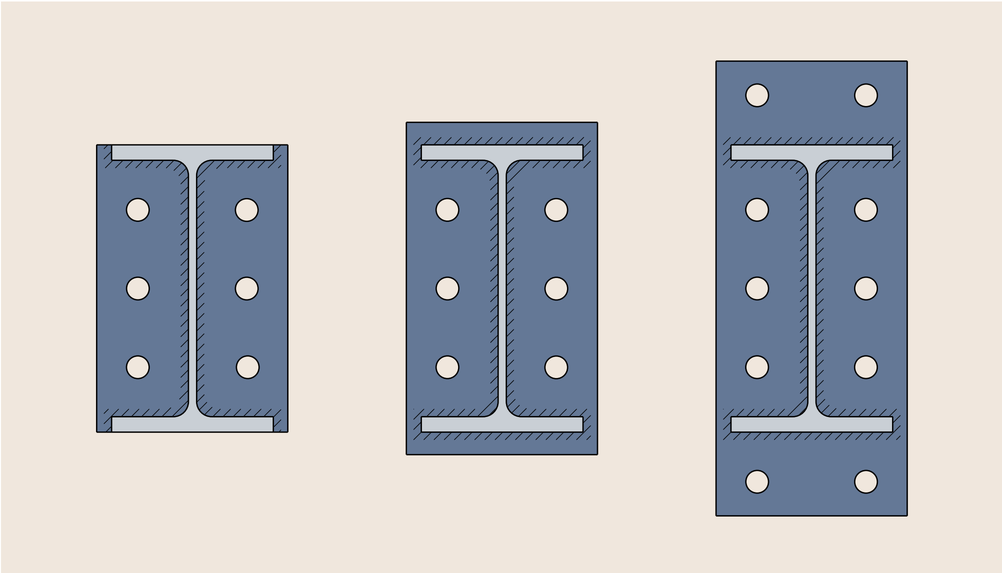


Figure 7.1 Types of end plate connections suitable for torsional resistance

7.2 Choice of end plate thickness

It is unlikely that the choice of end plate thickness would be governed by the torsional moment at the connection. The thickness required for the design of the connection as a nominally pinned or moment-resisting connection will normally be suitable for resistance to torsion combined with the other effects.

Guidance on the design of nominally pinned end plate connections is given in SCI publication P358, *Joints in steel construction – Simple connections (Eurocode version)*[17]. That publication advises that, for nominally pinned joints, end plates of 10 mm or 12 mm should be used. End plates of this thickness, with reasonable edge and end distances, will be adequate in most situations.

For moment-resisting connections (see SCI publication P207/95 *Joints in steel construction – Moment connections*[18]), an end plate thickness approximately equal to the bolt diameter is appropriate (i.e. 20 mm thick with M20 bolts, 25 mm thick with M24 bolts). It should, however, be recognized that the tension developed in the upper bolts will reduce available shear resistance.

7.3 Design resistance of end plate connections to combined shear and torsion

It is possible, but not altogether desirable, to lay down hard and fast rules for distribution of force among the bolts of a group resisting torsion in an end plate connection. A commonly adopted approach is to use a quasi-elastic calculation in which force in each bolt is proportional to its radial distance from a notional centre of rotation. Although the real force distribution will not match this calculation exactly, the divergence is no cause for concern. In reality, initial bolt forces will owe much to how perfectly (or otherwise) the holes align and there is considerable scope for plastic redistribution as the bolts bed themselves into the softer and more resilient plates they pass through.

In practice, a reasonable bolt force distribution is a matter for designer judgement. Equilibrium must always be satisfied. Outer bolts should not be expected to resist less force than inner ones, and a cautious view is advisable where the connection is (by accident or design) moment resisting. Upper bolt rows are liable to attract a certain amount of tension, which will reduce the available shear resistance (see Table 3.4 of BS EN 1993-1-8[19]).

A simple option for bolt forces due to torque on the end plate, with four symmetrically located bolts, is for each to be assigned one quarter of the torque divided by its radial distance from the centre of the group. Vertical shear can be added vectorially and the resultant compared with shear and bearing resistances in the normal way.

An alternative approach is to assume that the combined torsional moment and shear force is similar to the case of a shear force acting at an eccentricity from the centroid on the beam, as shown in Figure 7.2. In this approach, the design procedure may be

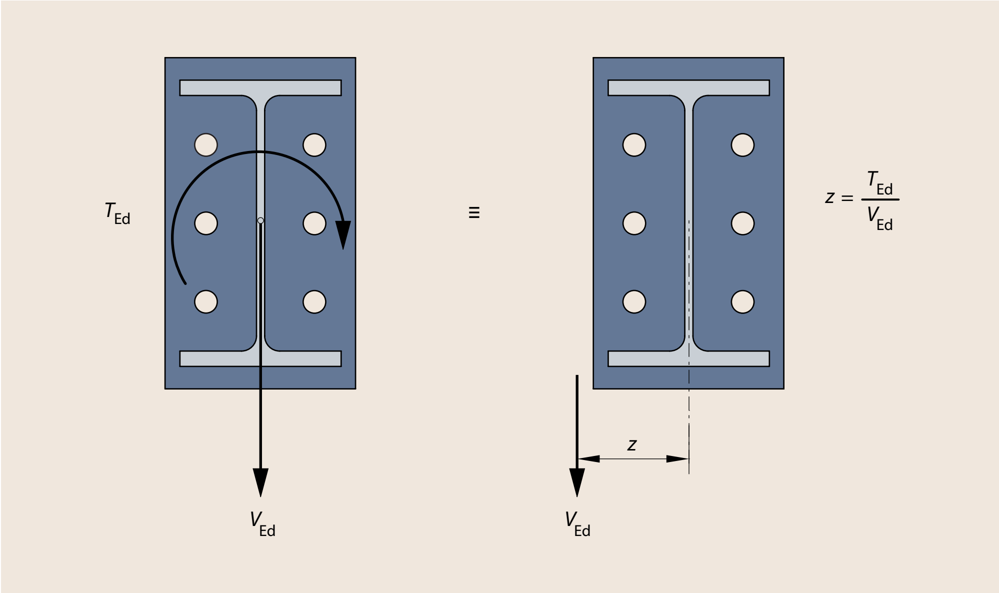


Figure 7.2 Alternative model for determining connection resistance to combined torsional moment and shear force

considered to be similar to that for a fin plate connection with two vertical lines of bolts. The procedure is described in SCI publication P358[17].

7.4 Bolt slip

Although there will be little in-plane distortion of the end plate itself, normal 2 mm oversize bolt holes inevitably permit some rotation of the beam. If this is a concern for serviceability reasons, an obvious solution would be to specify preloaded bolts.

7.5 The effect of bolt tension on shear resistance

Moment resisting connections with shear, both vertical and torsion-induced, should be verified, using the interaction criterion in Table 3.4 of BS EN 1993-1-8 (this allows a bolt fully utilized in tension to resist a coexisting shear force of up to 28% of its shear resistance).

7.6 Restraint against warping at member ends

It is important to recognize that a conventional moment connection, even one with a thick and extended end plate, does not provide warping fixity. For this purpose it is required that both top and bottom flanges are prevented from rotating (in opposite directions) in the plan view. Surrounding structure is unlikely to be in a position to provide such restraint, so an approach that has been put forward is to connect the two flanges rigidly together so that they may react against one another.

The details shown in Figure 7.3 illustrate two ways in which warping restraint might be provided, although such details will rarely be practical and cost-effective. Warping restraint can also be provided by casting a length of beam in a thick wall, though this too will rarely be a practical option.

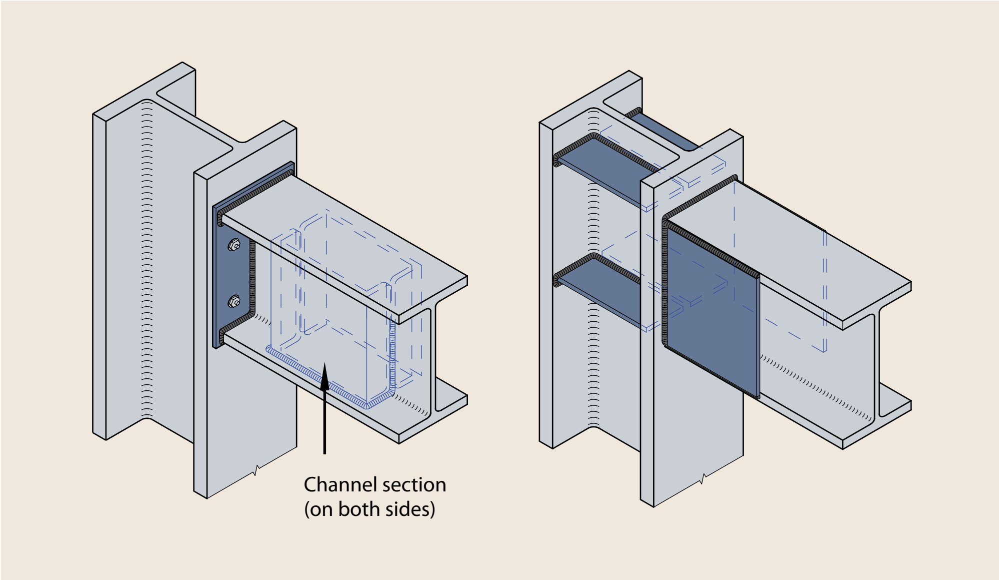


Figure 7.3 Warping fixity demands prevention of flange contra-rotation in the plan view

While it is usually preferable not to rely on warping fixity, there is no need to take active steps to avoid it. Any warping restraint can only increase torsional resistance and reduce rotation.

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APPENDIX A: TORSIONAL PROPERTIES OF SECTIONS - TABLES

These tables supplement the section properties in SCI P363[10] and other publications, whose coverage is not comprehensive for torsion calculations which involve warping. Additional properties tabulated here include Torsional Bending Constant, Normalized Warping Function(s) and Warping Statical Moment(s). Dimensions to the centroid and shear centre are given for non-doubly symmetric sections. All values are rounded to three significant figures.

The following properties are tabulated.

	Open sections				Hollow sections			
	UKB	UKC	PFC	ASB	Circular	Elliptical	Square	Rectangular
ST VENANT TORSIONAL CONSTANT	✓	✓	✓	✓	✓	✓	✓	✓
WARPING CONSTANT	✓	✓	✓	✓				
TORSIONAL BENDING CONSTANT	✓	✓	✓	✓				
NORMALIZED WARPING FUNCTION	✓	✓	✓					
WARPING STATICAL MOMENT	✓	✓	✓					
TORSIONAL SECTION MODULUS					✓	✓	✓	✓
LOCATION OF SHEAR CENTRE			✓	✓				

The properties needed for torsion calculations for hollow sections (I_T and W_t) are available in brochures and handbooks, but tables are included here for ease of reference. For square and rectangular hollow sections the properties are based on the corner geometry of Celsius® and similar hot finished sections. Properties for cold-formed corner geometry differ very slightly.

Properties for obsolete sections with tapering flanges (channels and joists) can be found in P057[5].

Appendix B gives information on methods of calculating section properties.

Table A.1 Universal beams (UKB) - Torsional Properties

SECTION DESIGNATION	MASS PER METRE	ST VENANT TORSIONAL CONSTANT	TORSIONAL BENDING CONSTANT	WARPING CONSTANT	NORMALIZED WARPING FUNCTION	WARPING STATICAL MOMENT
		I_T	a	I_w	W_{n0}	S_{w1}
	kg/m	cm⁴	m	dm⁶	cm²	cm⁴
1016 x 305	487	4300	1.97	64.4	758	31600
	437	3180	2.13	55.9	746	27900
	393	2330	2.32	48.4	736	24500
	349	1720	2.56	43.3	731	22100
	314	1260	2.78	37.6	723	19500
	272	835	3.16	32.2	719	16700
	249	582	3.46	26.9	716	14000
	222	390	3.78	21.5	712	11300
914 x 419	388	1730	3.64	88.8	930	35800
	343	1190	4.06	75.9	920	30800
914 x 305	289	926	2.96	31.2	688	16900
	253	626	3.30	26.4	680	14500
	224	422	3.68	22.0	674	12200
	201	291	4.04	18.4	669	10300
838 x 292	227	514	3.13	19.4	605	11900
	194	306	3.59	15.2	599	9500
	176	221	3.90	13.0	595	8160
762 x 267	197	404	2.69	11.3	499	8490
	173	267	3.02	9.39	494	7110
	147	159	3.47	7.40	488	5670
	134	119	3.75	6.46	486	4970
686 x 254	170	308	2.50	7.42	428	6490
	152	220	2.75	6.42	424	5670
	140	169	2.96	5.72	421	5080
	125	116	3.27	4.79	419	4290
610 x 305	238	785	2.18	14.4	471	11500
	179	340	2.78	10.1	458	8300
	149	200	3.26	8.18	452	6780
610 x 229	140	216	2.19	3.99	342	4360
	125	154	2.41	3.45	339	3810
	113	111	2.64	2.99	337	3320
	101	77.0	2.91	2.51	334	2820
610 x 178	100	95.0	1.99	1.45	264	2040
	92	71.0	2.13	1.24	263	1760
	82	48.8	2.35	1.04	261	1480
533 x 312	273	1290	1.74	15.0	432	13000
	219	642	2.11	11.0	422	9770
	182	373	2.47	8.79	414	7950
	151	216	2.90	7.03	408	6460
	kg/m	$\times 10^{-8}$ m⁴	m	$\times 10^{-6}$ m⁶	$\times 10^{-4}$ m²	$\times 10^{-8}$ m⁴

Table A.1 Universal beams (UKB) - Torsional Properties (continued)

SECTION DESIGNATION	MASS PER METRE	ST VENANT TORSIONAL CONSTANT	TORSIONAL BENDING CONSTANT	WARPING CONSTANT	NORMALIZED WARPING FUNCTION	WARPING STATICAL MOMENT
		I_T	a	I_w	W_{n0}	S_{w1}
	kg/m	cm⁴	m	dm⁶	cm²	cm⁴
533 x 210	138	250	1.66	2.67	281	3550
	122	178	1.84	2.32	277	3130
	109	126	2.02	1.99	274	2720
	101	101	2.16	1.81	273	2490
	92	76	2.34	1.60	271	2210
	82	52	2.59	1.33	269	1850
533 x 165	85	73.8	1.73	0.85	215	1480
	75	47.9	1.93	0.69	214	1210
	66	32.0	2.14	0.57	212	997
457 x 191	161	515	1.06	2.25	229	3660
	133	292	1.24	1.73	223	2890
	106	146	1.50	1.27	218	2170
	98	121	1.59	1.18	216	2040
	89	90.7	1.72	1.04	214	1820
	82	69.2	1.86	0.922	212	1620
	74	51.8	2.02	0.818	211	1450
	67	37.1	2.22	0.705	209	1260
457 x 152	82	89.2	1.31	0.592	174	1270
	74	65.9	1.43	0.518	172	1130
	67	47.7	1.56	0.448	170	982
	60	33.8	1.72	0.387	169	858
	52	21.4	1.94	0.311	167	694
406 x 178	85	93.0	1.43	0.728	181	1500
	74	62.8	1.58	0.608	178	1280
	67	46.1	1.73	0.533	177	1130
	60	33.3	1.90	0.466	175	997
	54	23.1	2.10	0.392	174	843
406 x 140	53	29.0	1.48	0.246	141	652
	46	19.0	1.68	0.207	139	555
	39	10.7	1.94	0.155	138	421
356 x 171	67	55.7	1.38	0.412	151	1020
	57	33.4	1.60	0.330	149	831
	51	23.8	1.76	0.286	147	726
	45	15.8	1.97	0.237	146	606
356 x 127	39	15.1	1.34	0.105	108	364
	33	8.79	1.55	0.081	107	284
305 x 165	54	34.8	1.32	0.234	124	708
	46	22.2	1.51	0.195	122	597
	40	14.7	1.70	0.164	121	509
	kg/m	$\times10^{-8}$ m⁴	m	$\times10^{-6}$ m⁶	$\times10^{-4}$ m²	$\times10^{-8}$ m⁴

Table A.1 Universal beams (UKB) - Torsional Properties (continued)

SECTION DESIGNATION	MASS PER METRE	ST VENANT TORSIONAL CONSTANT	TORSIONAL BENDING CONSTANT	WARPING CONSTANT	NORMALIZED WARPING FUNCTION	WARPING STATICAL MOMENT
		I_T	a	I_w	W_{n0}	S_{w1}
	kg/m	cm^4	m	dm^6	cm^2	cm^4
305 x 127	48	31.8	0.91	0.102	93.0	408
	42	21.1	1.02	0.085	91.7	345
	37	14.8	1.13	0.072	90.6	299
305 x 102	33	12.2	0.97	0.0442	77.3	214
	28	7.40	1.11	0.0349	76.3	171
	25	4.77	1.22	0.0273	75.7	135
254 x 146	43	23.9	1.06	0.103	90.9	425
	37	15.3	1.20	0.0858	89.7	358
	31	8.55	1.41	0.0660	88.7	279
254 x 102	28	9.57	0.87	0.0281	64.0	163
	25	6.42	0.97	0.0231	63.4	136
	22	4.15	1.07	0.0182	62.8	108
203 x 133	30	10.3	0.97	0.0374	66.0	212
	25	5.96	1.13	0.0294	65.1	169
203 x 102	23	7.02	0.75	0.0154	49.3	117
178 x 102	19	4.41	0.76	0.0099	43.0	85.9
152 x 89	16	3.56	0.59	0.0047	32.1	54.8
127 x 76	13	2.85	0.43	0.0020	22.7	32.8
	kg/m	$\times 10^{-8}\ m^4$	m	$\times 10^{-6}\ m^6$	$\times 10^{-4}\ m^2$	$\times 10^{-8}\ m^4$

Table A.2 Universal columns (UKC) – Torsional properties

SECTION DESIGNATION	MASS PER METRE	ST VENANT TORSIONAL CONSTANT	TORSIONAL BENDING CONSTANT	WARPING CONSTANT	NORMALIZED WARPING FUNCTION	WARPING STATICAL MOMENT
		I_T	a	I_w	W_{n0}	S_{w1}
	kg/m	cm⁴	m	dm⁶	cm²	cm⁴
356 x 406	634	13700	0.86	38.8	421	34400
	551	9240	0.93	31.1	406	28700
	467	5810	1.04	24.3	390	23300
	393	3550	1.18	18.9	376	18800
	340	2340	1.31	15.5	366	15800
	287	1440	1.49	12.3	356	13000
	235	812	1.75	9.54	346	10300
356 x 368	202	558	1.82	7.16	326	8240
	177	381	2.03	6.08	321	7110
	153	251	2.30	5.13	316	6060
	129	153	2.66	4.17	312	5020
305 x 305	283	2030	0.899	6.34	259	9190
	240	1271	1.01	5.03	251	7520
	198	734	1.17	3.88	243	5990
	158	378	1.40	2.87	235	4570
	137	249	1.58	2.39	231	3870
	118	161	1.79	1.98	227	3270
	97	91	2.11	1.56	223	2620
254 x 254	167	626	0.823	1.63	171	3590
	132	319	0.982	1.19	164	2710
	107	172	1.16	0.899	159	2110
	89	102	1.35	0.717	156	1730
	73	58	1.59	0.563	153	1380
203 x 203	128	427	0.578	0.549	113	1820
	114	305	0.628	0.464	110	1570
	100	210	0.691	0.386	108	1340
	86	137	0.777	0.318	105	1130
	71	80.2	0.899	0.250	102	914
	60	47.2	1.04	0.197	101	734
	52	31.8	1.17	0.167	98.9	632
	46	22.2	1.29	0.143	97.8	548
152 x 152	51	48.8	0.568	0.061	60.8	376
	44	31.7	0.639	0.050	59.4	315
	37	19.2	0.734	0.040	58.0	258
	30	10.5	0.871	0.031	56.6	204
	23	4.63	1.09	0.021	55.4	143
	kg/m	$\times 10^{-8}$ m⁴	m	$\times 10^{-6}$ m⁶	$\times 10^{-4}$ m²	$\times 10^{-8}$ m⁴

Table A.3 Parallel Flange Channels (PFC) – Torsional properties

SECTION DESIGNATION	MASS PER METRE	ST VENANT TORSIONAL CONSTANT	TORSIONAL BENDING CONSTANT	WARPING CONSTANT	NORMALIZED WARPING FUNCTION		WARPING STATICAL MOMENT			DISTANCE FROM CENTRE OF WEB TO SHEAR CENTRE	DISTANCE FROM SHEAR CENTRE TO CENTROID
		I_T	a	I_w	W_{n0}	W_{n2}	S_{w1}	S_{w2}	S_{w3}	e_0	$e_{s,c}$
	kg/m	cm^4	m	dm^6	cm^2	cm^2	cm^4	cm^4	cm^4	mm	mm
430 x 100	64.4	65.7	0.916	0.219	127.0	67.2	780	561	491	32.7	53.4
380 x 100	54	47.6	0.893	0.150	109.6	63.0	608	407	347	34.8	57.9
300 x 100	46	38.4	0.729	0.0813	83.3	52.1	427	260	205	36.7	62.7
300 x 90	41	30.0	0.699	0.0581	76.5	45.2	336	219	178	31.8	53.3
260 x 90	35	21.4	0.669	0.0379	65.0	40.7	254	155	120	33.1	56.5
260 x 75	28	12.2	0.649	0.0203	56.3	32.3	161	108	90	26.1	43.6
230 x 90	32	20.1	0.590	0.0279	55.9	37.3	215	120	86	34.5	60.0
230 x 75	26	12.3	0.558	0.0153	47.9	30.1	140	84.6	67.3	27.7	47.4
200 x 90	30	19.1	0.508	0.0197	47.0	33.4	179	88.4	56.7	36.0	63.7
200 x 75	23	11.6	0.481	0.0107	40.3	27.2	115	62.5	46.3	29.1	50.9
180 x 90	26	13.8	0.506	0.0141	42.3	30.4	143	69.5	40.8	36.3	64.7
180 x 75	20	7.60	0.498	0.00754	36.8	24.2	89.3	50.8	35.8	28.5	49.6
150 x 90	24	12.2	0.426	0.00890	34.3	25.6	111.1	49.3	20.9	37.1	66.8
150 x 75	18	6.31	0.429	0.00467	29.8	20.8	67.9	34.8	21.0	29.7	52.8
125 x 65	15	4.89	0.313	0.00194	21.3	14.7	40.3	21.1	12.1	25.4	45.2
100 x 50	10	2.64	0.215	0.000491	12.9	8.83	16.9	8.99	5.21	19.3	34.1
	kg/m	$\times 10^{-8}$ m^4	m	$\times 10^{-6}$ m^6	$\times 10^{-4}$ m^2	$\times 10^{-4}$ m^2	$\times 10^{-8}$ m^4	$\times 10^{-8}$ m^4	$\times 10^{-8}$ m^4	$\times 10^{-3}$ m	mm

Note that the values of the St Venant Torsional Constant are slightly higher than those in P363. The tabulated values have been calculated in accordance with the more exact expressions in Appendix B.

Table A.4 Asymmetric Slimflor® Beams (ASB)– Torsional properties

SECTION DESIGNATION	MASS PER METRE	ST VENANT TORSIONAL CONSTANT	TORSIONAL BENDING CONSTANT	WARPING CONSTANT	DISTANCE FROM CENTROID TO SHEAR CENTRE	DISTANCE FROM BOTTOM FLANGE CENTRE TO SHEAR CENTRE	DISTANCE FROM TOP FLANGE CENTRE TO SHEAR CENTRE	TOP FLANGE PLASTIC SECTION MODULUS
		I_T	a	I_w	$e_{s,c}$	$e_{s,bf}$	$e_{s,tf}$	$W_{pl,z,tf}$
	kg/m	cm^4	m	dm^6	mm	mm	mm	cm^3
300 ASB	249	2000	0.508	2.00	65	65	237	412
	196	1180	0.574	1.50	65	59	243	335
	185	871	0.599	1.20	65	60	231	276
	155	620	0.668	1.07	64	56	238	256
	153	513	0.672	0.895	66	58	228	217
280 ASB	136	379	0.696	0.710	61	54	212	199
	124	332	0.751	0.721	59	52	218	206
	105	207	0.847	0.574	59	50	216	170
	100	160	0.854	0.451	61	51	209	135
	74	72.2	1.101	0.338	59	49	209	107
	kg/m	$\times 10^{-8}\ m^4$	m	$\times 10^{-6}\ m^6$	$\times 10^{-3}$ m	$\times 10^{-3}$ m	$\times 10^{-3}$ m	$\times 10^{-6}\ m^3$

Table A.5 Circular hollow sections – Torsional properties

SECTION DESIGNATION	MASS PER METRE	ST VENANT TORSIONAL CONSTANT	TORSIONAL SECTION MODULUS
		I_T	W_t
	kg/m	cm⁴	cm³
21.3 x 3.2	1.43	1.54	1.44
26.9 x 3.2	1.87	3.41	2.53
33.7 x 3.2	2.41	7.21	4.28
33.7 x 4	2.93	8.38	4.97
42.4 x 3.2	3.09	15.2	7.19
42.4 x 4	3.79	18.0	8.48
48.3 x 3.2	3.56	23.2	9.59
48.3 x 4	4.37	27.5	11.4
48.3 x 5	5.34	32.3	13.4
48.3 x 6.3	6.53	37.5	15.5
60.3 x 3.2	4.51	46.9	15.6
60.3 x 4	5.55	56.3	18.7
60.3 x 5	6.82	67.0	22.2
60.3 x 6.3	8.39	79.0	26.2
76.1 x 2.9	5.24	89.5	23.5
76.1 x 3.2	5.75	97.6	25.6
76.1 x 4	7.11	118	31.0
76.1 x 5	8.77	142	37.3
76.1 x 6.3	10.8	170	44.6
76.1 x 8	13.4	201	52.9
88.9 x 4	8.38	193	43.3
88.9 x 5	10.3	233	52.4
88.9 x 6.3	12.8	280	63.1
88.9 x 8	16	336	75.6
114.3 x 3.6	9.83	384	67.2
114.3 x 4	10.9	422	73.9
114.3 x 5	13.5	514	89.9
114.3 x 6.3	16.8	625	109
114.3 x 8	21	759	133
139.7 x 5	16.6	961	138
139.7 x 6.3	20.7	1180	169
139.7 x 8	26	1440	206
139.7 x 10	32	1720	247
168.3 x 5	20.1	1710	203
168.3 x 6.3	25.2	2110	250
168.3 x 8	31.6	2590	308
168.3 x 10	39	3130	372
168.3 x 12.5	48	3740	444
	kg/m	$\times 10^{-8}$ m⁴	$\times 10^{-6}$ m³

SECTION DESIGNATION	MASS PER METRE	ST VENANT TORSIONAL CONSTANT	TORSIONAL SECTION MODULUS
		I_T	W_t
	kg/m	cm⁴	cm³
193.7 x 5	23.3	2640	273
193.7 x 6.3	29.1	3260	337
193.7 x 8	36.6	4030	416
193.7 x 10	45.3	4880	504
193.7 x 12.5	55.9	5870	606
219.1 x 6.3	33.1	4770	436
219.1 x 8	41.6	5920	540
219.1 x 10	51.6	7200	657
219.1 x 12.5	63.7	8690	793
219.1 x 16	80.1	10600	967
244.5 x 12.5	71.5	12300	1006
244.5 x 16	90.2	15100	1200
273 x 10	64.9	14300	1000
273 x 12.5	80.3	17400	1300
273 x 16	101	21400	1600
323.9 x 10	77.4	24300	1500
323.9 x 12.5	96	29700	1800
323.9 x 16	121	36800	2300
355.6 x 16	134	49300	2800
406.4 x 10	97.8	49000	2400
406.4 x 16	154	74900	3700
457 x 10	110	70200	3100
457 x 16	174	108000	4700
508 x 12.5	153	120000	4700
508 x 16	194	150000	5900
	kg/m	$\times 10^{-8}$ m⁴	$\times 10^{-6}$ m³

Table A.6 Elliptical hollow sections – Torsional properties

SECTION DESIGNATION	MASS PER METRE	ST VENANT TORSIONAL CONSTANT	TORSIONAL SECTION MODULUS
		I_T	W_t
	kg/m	cm^4	cm^3
300 x 150 x 12.5	65.5	7050	686
400 x 200 x 12.5	88.6	17600	1300
400 x 200 x 16.0	112	21600	1580
	kg/m	$\times 10^{-8}$ m^4	$\times 10^{-6}$ m^3

Table A.7 Square hollow sections – Torsional properties

SECTION DESIGNATION	MASS PER METRE	ST VENANT TORSIONAL CONSTANT	TORSIONAL SECTION MODULUS
		I_T	W_t
	kg/m	cm⁴	cm³
40 x 40 x 3.2	3.61	16.5	7.42
40 x 40 x 4	4.39	19.5	8.54
40 x 40 x 5	5.28	22.5	9.60
50 x 50 x 3.2	4.62	33.8	12.4
50 x 50 x 4	5.64	40.4	14.5
50 x 50 x 5	6.85	47.6	16.7
50 x 50 x 6.3	8.31	55.2	18.8
60 x 60 x 3.2	5.62	60.2	18.6
60 x 60 x 4	6.9	72.5	22.0
60 x 60 x 5	8.42	86.4	25.7
60 x 60 x 6.3	10.3	102	29.6
60 x 60 x 8	12.5	118	33.4
70 x 70 x 3.2	6.63	98	26.1
70 x 70 x 4	8.15	118	31.2
70 x 70 x 5	9.99	142	36.8
70 x 70 x 6.3	12.3	169	42.9
70 x 70 x 8	15	200	49.2
80 x 80 x 3.2	7.63	148	34.9
80 x 80 x 4	9.41	180	41.9
80 x 80 x 5	11.6	217	49.8
80 x 80 x 6.3	14.2	262	58.7
80 x 80 x 8	17.5	312	68.3
90 x 90 x 4	10.7	260	54.2
90 x 90 x 5	13.1	316	64.8
90 x 90 x 6.3	16.2	382	77.0
90 x 90 x 8	20.1	459	90.5
100 x 100 x 3.6	10.8	328	62.3
100 x 100 x 4	11.9	361	68.2
100 x 100 x 5	14.7	439	81.8
100 x 100 x 6.3	18.2	534	97.8
100 x 100 x 8	22.6	646	116
100 x 100 x 10	27.4	761	133
120 x 120 x 5	17.8	777	122
120 x 120 x 6.3	22.2	950	147
120 x 120 x 8	27.6	1160	176
120 x 120 x 10	33.7	1380	206
120 x 120 x 12.5	40.9	1620	236
140 x 140 x 5	21	1250	170
140 x 140 x 6.3	26.1	1540	206
140 x 140 x 8	32.6	1890	249
140 x 140 x 10	40	2270	294
140 x 140 x 12.5	48.7	2700	342
	kg/m	$\times 10^{-8}$ m⁴	$\times 10^{-6}$ m³

SECTION DESIGNATION	MASS PER METRE	ST VENANT TORSIONAL CONSTANT	TORSIONAL SECTION MODULUS
		I_T	W_t
	kg/m	cm⁴	cm³
150 x 150 x 5	22.6	1550	197
150 x 150 x 6.3	28.1	1910	240
150 x 150 x 8	35.1	2350	291
150 x 150 x 10	43.1	2830	344
150 x 150 x 12.5	52.7	3370	402
160 x 160 x 6.3	30.1	2330	275
160 x 160 x 8	37.6	2880	335
160 x 160 x 10	46.3	3480	398
160 x 160 x 12.5	56.6	4160	467
180 x 180 x 6.3	34	3360	355
180 x 180 x 8	42.7	4160	434
180 x 180 x 10	52.5	5050	518
180 x 180 x 12.5	64.4	6070	613
180 x 180 x 16	80.2	7340	724
200 x 200 x 5	30.4	3760	362
200 x 200 x 6.3	38	4650	444
200 x 200 x 8	47.7	5780	545
200 x 200 x 10	58.8	7030	655
200 x 200 x 12.5	72.3	8490	778
200 x 200 x 16	90.3	10300	927
250 x 250 x 6.3	47.9	9240	712
250 x 250 x 8	60.3	11500	880
250 x 250 x 10	74.5	14100	1060
250 x 250 x 12.5	91.9	17200	1280
250 x 250 x 16	115	21100	1550
300 x 300 x 8	72.8	20200	1290
300 x 300 x 10	90.2	24800	1580
300 x 300 x 12.5	112	30300	1900
300 x 300 x 16	141	37600	2330
350 x 350 x 8	85.4	32400	1790
350 x 350 x 10	106	39900	2190
350 x 350 x 12.5	131	48900	2650
350 x 350 x 16	166	61000	3260
400 x 400 x 10	122	60100	2900
400 x 400 x 12.5	151	73900	3530
400 x 400 x 16	191	92400	4360
400 x 400 x 20	235	112000	5240
	kg/m	$\times 10^{-8}$ m⁴	$\times 10^{-6}$ m³

Table A.8 Rectangular hollow sections – Torsional properties

SECTION DESIGNATION	MASS PER METRE	ST VENANT TORSIONAL CONSTANT	TORSIONAL SECTION MODULUS
		I_T	W_t
	kg/m	cm⁴	cm³
50 x 30 x 3.2	3.61	14.2	6.80
50 x 30 x 4	4.39	16.6	7.77
50 x 30 x 5	5.28	19.0	8.67
60 x 40 x 3.2	4.62	30.8	11.7
60 x 40 x 4	5.64	36.7	13.7
60 x 40 x 5	6.85	43.0	15.7
60 x 40 x 6.3	8.31	49.5	17.6
80 x 40 x 3.2	5.62	46.2	16.1
80 x 40 x 4	6.9	55.2	18.9
80 x 40 x 5	8.42	65.1	21.9
80 x 40 x 6.3	10.3	75.6	24.8
80 x 40 x 8	12.5	85.8	27.4
90 x 50 x 3.2	6.63	80.9	23.6
90 x 50 x 4	8.15	97.5	28.0
90 x 50 x 5	9.99	116	32.9
90 x 50 x 6.3	12.3	138	38.1
90 x 50 x 8	15	160	43.2
100 x 50 x 3.2	7.13	93	26.4
100 x 50 x 4	8.78	113	31.4
100 x 50 x 5	10.8	135	36.9
100 x 50 x 6.3	13.3	160	42.9
100 x 50 x 8	16.3	186	48.9
100 x 60 x 3.2	7.63	129	32.4
100 x 60 x 4	9.41	156	38.7
100 x 60 x 5	11.6	188	45.9
100 x 60 x 6.3	14.2	224	53.8
100 x 60 x 8	17.5	265	62.2
120 x 60 x 4	10.7	201	47.1
120 x 60 x 5	13.1	242	56.0
120 x 60 x 6.3	16.2	290	65.9
120 x 60 x 8	20.1	344	76.6
120 x 80 x 4	11.9	330	65.0
120 x 80 x 5	14.7	401	77.9
120 x 80 x 6.3	18.2	487	92.9
120 x 80 x 8	22.6	587	110
120 x 80 x 10	27.4	688	126
150 x 100 x 5	18.6	807	127
150 x 100 x 6.3	23.1	986	153
150 x 100 x 8	28.9	1200	183
150 x 100 x 10	35.3	1430	214
150 x 100 x 12.5	42.8	1680	246
160 x 80 x 5	17.8	600	106
160 x 80 x 6.3	22.2	730	127
160 x 80 x 8	27.6	880	151
160 x 80 x 10	33.7	1040	175
160 x 80 x 12.5	40.9	1200	198
200 x 100 x 5	22.6	1200	172
200 x 100 x 6.3	28.1	1470	208
200 x 100 x 8	35.1	1800	251
200 x 100 x 10	43.1	2160	295
200 x 100 x 12.5	52.7	2540	341
200 x 120 x 6.3	30.1	2030	255
200 x 120 x 8	37.6	2490	310
200 x 120 x 10	46.3	3000	367
200 x 150 x 8	41.4	3640	398
200 x 150 x 10	51	4410	475
250 x 150 x 6.3	38	4050	413
250 x 150 x 8	47.7	5020	506
250 x 150 x 10	58.8	6090	605
250 x 150 x 12.5	72.3	7330	717
250 x 150 x 16	90.3	8870	849
300 x 100 x 8	47.7	3070	387
300 x 100 x 10	58.8	3680	458
300 x 200 x 6.3	47.9	8480	681
300 x 200 x 8	60.3	10600	840
300 x 200 x 10	74.5	12900	1020
300 x 200 x 12.5	91.9	15700	1220
300 x 200 x 16	115	19300	1470
300 x 250 x 8	66.5	15200	1070
340 x 100 x 10	65.1	4300	523
400 x 150 x 16	128	16800	1430
400 x 200 x 8	72.8	15700	1130
400 x 200 x 10	90.2	19300	1380
400 x 200 x 12.5	112	23400	1660
400 x 200 x 16	141	28900	2010
450 x 250 x 8	85.4	27100	1630
450 x 250 x 10	106	33300	1990
450 x 250 x 12.5	131	40700	2410
450 x 250 x 16	166	50500	2950
500 x 300 x 10	122	52400	2700
500 x 300 x 16	191	80300	4040
500 x 300 x 20	235	97400	4840
	kg/m	$\times 10^{-8}$ m⁴	$\times 10^{-6}$ m³

APPENDIX B: TORSIONAL PROPERTIES OF SECTIONS-FORMULAE

This Appendix provides calculation methods for some torsional properties of commonly used structural shapes. It records how the torsional properties given in the Tables in Appendix A were derived, and may be useful for manual calculation of the properties of other rolled or fabricated sections.

General expressions for section properties may be found in textbooks, for example *Theory of Elasticity (Engineering societies monographs)*[6]. Software such as *LTBeam* is also available to generate properties of a given section.

B.1 Shear centre location

To determine the location of the shear centre of a section composed of thin elements, the elements may be represented by their centrelines. For simplicity, the root radii for rolled sections may be neglected.

Where a shear force V acts on a section, and it is not to twist, the couple developed by equal and opposite shear forces in the flanges, acting as a couple, must be balanced by torque Ve in which e is the eccentricity from the line of shear force in the (vertical) web. This principle can be applied both to channel sections and ASB sections. For doubly symmetric and monosymmetric sections, $e = 0$.

B.1.1 Shear centre in parallel flange channels

For an equal flanged channel section, the element lengths are equal to $(b - t_w/2)$ for the flanges and $(h - t_f)$ for the web. By symmetry, the shear centre is located on the major axis, as shown in Figure 4.2.

To determine the horizontal position of the shear centre, consider the shear forces in the flanges associated with bending about the major axis (these can be determined using the familiar $VA\bar{z}/I_y$ expressions for shear flow at a location).

The total shear force F_f acting horizontally in each flange is given by:

$$F_f \quad = ½ \{V \times t_f (b - t_w/2) \times [(h - t_f)/2]/I_y\} \times (b - t_w/2)$$

The couple due to these forces is then $F_f \times (h - t_f)$, which must be balanced by $V \times e_0$.

Hence the eccentricity of the shear centre relative to the centreline of the web is given by:

$$e_0 = t_f (b - t_w/2)^2 (h - t_f)^2/(4I_y)$$

Note: the use of the symbol e_0 for this dimension is consistent with P363.

B.1.2 Shear centre in ASB sections

ASB sections are monosymmetric about their minor axis so the shear centre will be located on the centreline of the web. The centroid is also located on the centreline, above the shear centre. The principal dimensions of an ASB and the locations of the shear centre and centroid are illustrated in Figure B.1.

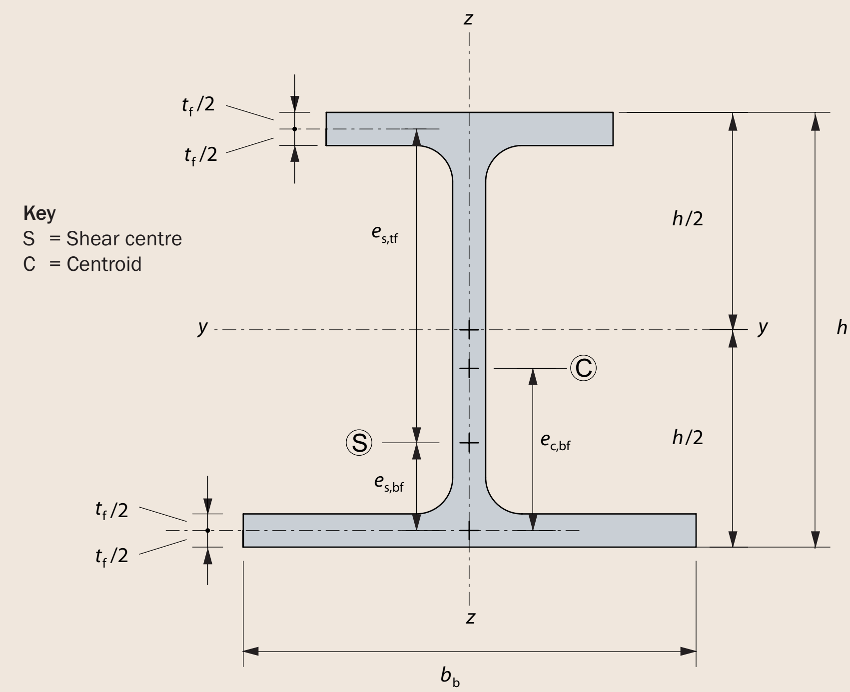


Figure B.1 ASB dimensions

To determine the location of the shear centre, consider shear forces in the two flanges that are proportional to each flange's second moment of area about the z-z axis: the location of the shear centre will be on the line of the resultant of the two forces. The location can thus be determined by taking moments of the two forces (actually, the I values) about any convenient location.

The shear centre location relative to the mid-thickness of the bottom flange is thus given by:

$$e_{s,bf} = (h - t_f)\, I_{z,tf}/(I_{z,tf} + I_{z,bf})$$

where $I_{z,tf}$ and $I_{z,bf}$ are the second moments of area of the top and bottom flanges respectively. The web and the fillets of the rolled section are assumed to have a negligible effect on the location of the shear centre and are thus excluded from the values of $I_{z,tf}$ and $I_{z,bf}$. (The thick web would make a small contribution to the second moment of area of the whole section about the z-z axis but its influence on the location of the shear centre is much less.)

In contrast, the location of the centroid is influenced by the area of the web and, to a lesser extent, by the fillets. To determine its position it is convenient to consider the ASB section in two parts - a bisymmentric I section with both flanges of width b_t and the two remaining portions of the bottom flange, each of width $(b_b - b_t)/2$.

The combined area of the two flange tips is given by:

$$A_1 \qquad = (b_b - b_t)t_f$$

The area of the bisymmetric portion is given by:

$$A_2 \qquad = A - A_1$$

where A is the area of the whole ASB cross section.

The position of the centroid, relative to the mid-thickness of the bottom flange is thus given by:

$$e_{c,bf} \qquad = [(h - t_f)/2]\,A_2/A$$

B.2 St Venant torsional constant I_T

B.2.1 Solid rectangles

The St Venent torsional constant for a rectangular section of dimensions $a \times b$ (where $a \geq b$) is given by:

$$I_T \qquad = \frac{ab^3}{3}\left[1 - 0.21\frac{b}{a}\left(1 - \frac{b^4}{12a^4}\right)\right]$$

For a long rectangle this simplifies to:

$$I_T \qquad = \frac{ab^3}{3} - 0.21b^4 \qquad \text{or, for } a >> b,\ I_T = \frac{ab^3}{3}$$

The deduction $0.21b^4$ represents a reduction at the ends of a rectangle.

B.2.2 Open sections

For typical structural sections composed of three (or, in the case of angles, two) rectangular elements whose thickness t is small relative to length L, a reasonable approximation may be obtained by summing $Lt^3/3$ for the rectangular elements. Thus, for a symmetric I section:

$I_T \approx [2bt_f^3 + (h - 2t_f)t_w^3]/3$

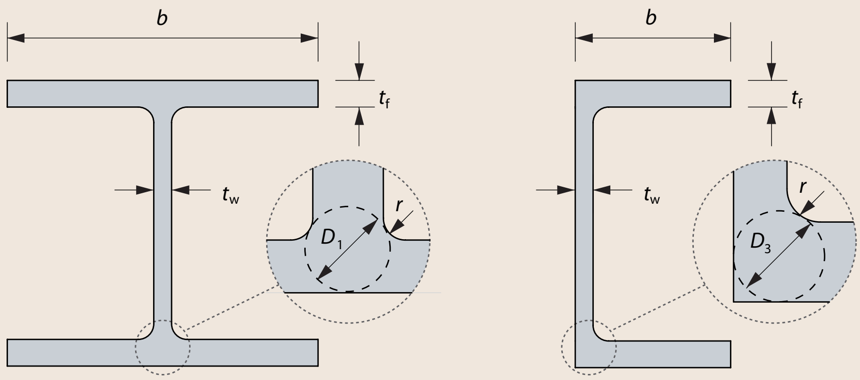
and for an ASB:

$I_T \approx [(b_t + b_b)t_f^3 + (h - 2t_f)t_w^3]/3$

For rolled sections, especially those with relatively large root radii, the degree of conservatism in the above expression can be significant. A more accurate assessment, correcting for the deductions at the open ends and the enhancement at the junction, is warranted. The methodology given below was developed by El Darwish & Johnston in the 1960s, as described in El Darwish and Johnston*.

The deduction at each end is $0.105t^4$, where t is the thickness of the element. When there are four flange tips, there are four deductions to be made; when there are only two flange tips only two deductions are made (the above reference mistakenly shows four deductions for channels).

The junction enhancement is αD^4, where D is the diameter of the largest circle that can be inscribed within the section and α is a dimensionless coefficient obtained from a graph or an empirical formula. There are different graphs and formulae for different junction geometries. The ones relevant to current (parallel flanged) sections are α_1 for T-junctions, as in I sections and α_3 for L-junctions as in channel and angle sections. For obsolete sections with tapering flanges, values may be found in El Darwish and Johnston or in P057[5].



$I_T = [2bt_f^3 + (h - 2t_f)t_w^3]/3$
$+ 2\alpha_1 D_1^4 - 4 \times 0.105t_f^4$

$I_T = [2bt_f^3 + (h - 2t_f)t_w^3]/3$
$+ 2\alpha_1 D_1^4 - 2 \times 0.105t_f^4$

Figure B.2 Junction corrections based on the inscribed circle

The empirical formulae are:

$\alpha_1 = -0.042 + 0.2204t_w/t_f + 0.1355r/t_f - 0.0865t_w r/t_f^2 - 0.0725t_w^2/t_f^2$

$\alpha_3 = -0.0908 + 0.2621t_w/t_f + 0.1231r/t_f - 0.0752t_w r/t_f^2 - 0.0945t_w^2/t_f^2$

* El Darwish, I.A. and Johnston, B.G., Torsion of structural shapes, ASCE Journal of the Structural Division, Volume 91, ST1, February 1965

The diameters of the inscribed circles are given by:

For T-junction $D_1 = [(t_f + r)^2 + t_w(r + t_w /4)]/(2r + t_f)$

For L-junction $D_3 = 2\{(3r + t_w + t_f) - \sqrt{[2(2r + t_w)(2r + t_f)]}\}$

The junction corrections will be additive and if there are two junctions there are two additions to be made.

B.2.3 Structural hollow sections

For any thin-walled hollow section of uniform thickness t, the value of I_T is closely approximated by:

$$I_T \quad = 4tA_p^2/p$$

in which p is the mean perimeter length and A_p is the area enclosed by the mean perimeter, which follows the centre of the tube wall.

A closer approximation, used in the Product Standard EN 10210-2:2006 (for hot finished structural hollow sections), expressed using the symbols for area and perimeter in this guide, is:

$$I_T \quad = 4tA_p^2/p + pt^3/3$$

The $pt^3/3$ term allows for the variation of shear stress across the thickness of the section. If the same section were to be converted into an open one by a longitudinal cut, this would be the value of its torsional constant.

For circular hollow sections, uniquely, an exact formulation is available:

$$I_T \quad = \pi[d^4 - (d - 2t)^4]/32 \ (= I_p = 2I)$$

For elliptical hollow sections, the length of the perimeter of an ellipse (expressed here as p) is given by EN 10210-2 (again using symbols in this guide) as:

$$p \quad = (\pi/2)(h + b - 2t)\{1 + 0.25[(h - b)/(h + b - 2t)]^2\}$$

The area enclosed is given as $A_p = \pi(h - t)(b - t)/4$.

For rectangular hollow sections, the perimeter length is:

$$p \quad = 2[(h - t) + (b - t)] - 2r(4 - \pi)$$

in which r is the ‘mean corner radius’. For hot finished sections, $r = 1.25t$, according to EN 10210-2; for cold formed sections the mean corner radius is between $1.5t$ and

$2.5t$, according to values for calculation of section properties given in Annex B of EN 10219-2:2006. (The properties for cold formed sections are therefore very slightly smaller than those for hot rolled sections. Only the values for hot rolled sections are given in Appendix A.)

For a rectangular hollow section, the area is given by:

$$A_p = (h - t)(b - t) - r^2(4 - \pi)$$

B.3 Warping constant I_w

B.3.1 I sections

Doubly symmetric (equal flanged) I sections

In a doubly symmetric section, the warping moment M_w is given by:

$$M_{w,Ed} = \pm EI_f \phi''(h - t_f)/2$$

and the warping torque is given by:

$$T_{w,Ed} = EI_f \phi'''(h - t_f)^2/2$$

where I_f is the second moment of area of one flange

I_w is therefore equal to $I_f(h - t_f)^2/2$

For practical purposes I_f may be taken as half of I_z for the section as a whole, so $I_w \approx I_z (h - t_f)^2/4$.

Monosymmetric (unequal flanged) I sections

Although the flanges are unequal, M_w will be numerically the same for both flanges and the product of I_f and its distance from the shear centre will be the same for both flanges. Therefore, based on the bottom flange:

$$M_{w,Ed} = EI_{bf} \phi'' e_{s,bf}$$

where

$e_{s,bf}$ is the height of the shear centre above the centre of the bottom flange

I_{bf} is the second moment of area of the bottom flange about the z axis.

The warping torsional moment is:

$$T_{w,Ed} = EI_{bf} \phi''' e_{s,bf} (h - t_f)$$

I_w is therefore equal to $EI_{bf}\, e_{s,bf}(h - t_f)$.

B.3.2 Parallel flange channels

For an equal-flanged channel section, L sections, each composed of a flange plus half the web, can act to resist warping (see Figure 4.3).

The value of the warping torsional constant is given by consideration of bending of the L sections about both z-z and y-y axes. From SCI publication P363, the value of I_w is given as:

$$I_w = \frac{(h-t_f)^2}{4}\left[I_z - A\left(c_z - t_w/2\right)^2\left(\frac{(h-t_f)^2 A}{4I_y} - 1\right)\right]$$

where c_z is the distance from the back of the web to the centroidal axis and A is the area of the cross section.

B.4 Torsional section modulus W_t

B.4.1 For an open section

For an open section, the torsional section modulus is given simply by:

$$W_t = I_T/t$$

Where

I_T is the St Venant torsional constant

t is the thickness at the point considered.

B.4.2 For a structural hollow section

For any thin-walled hollow section, W_t is given approximately as:

$$W_t = 2tA_p$$

where A_p is as defined in B.2.3 and t is the thickness of the section.

A more exact formula given in EN 10210-2 for rectangular, square and elliptical sections (using the symbols in this guide) is:

$$W_t = I_T/(t + 2A_p/p) = (4A_p^2 t/p + pt^3/3)/(t + 2A_p/p)$$

where I_T, t and r are as defined in B.2.3

For circular sections, the more exact formula is:

$$W_t = 4I/d$$

in which $I = \pi[d^4 - (d - t)^4]/64$.

APPENDIX C: SOLUTIONS FOR ϕ AND ITS DERIVATIVES - FORMULAE

The general equation for torsional moment is:

$$T / GI_T = \phi' - a^2\phi'''$$

where

$$a = \sqrt{EI_W / GI_T}$$

Solving this differential equation for the variation of ϕ with distance along the beam x, gives solutions of the form:

$$\phi = \text{A}\sinh(x/a) + \text{B}\cosh(x/a) + \text{C}x^3 + \text{D}x^2 + \text{E}x + \text{F}$$

where A to F are constants that depend on load distribution and end conditions.

The constants have been evaluated for ten cases. The expressions for ϕ, ϕ', ϕ'' and ϕ''' for these ten cases are given below. Each case is illustrated with a diagram that shows the form of loading and the end conditions.

The fork device in the explanatory sketches is to indicate that the member is prevented from twisting but not from warping at the point of restraint. Usually, but not invariably, this is also a point of support in the conventional sense. The jaws device is to indicate that the member is held in a vice-like grip that prevents contra-rotation of the flanges. This is the assumption of warping restraint which, as discussed in Sections 2.6 and 7.6, is difficult to realize in practice. If it can be achieved, it is more than likely that the member will find itself restrained against torsion and bending (both M_y and M_z) at the same point – the vice is attached to a bench, so to speak.

Continuity does not provide warping restraint. Nevertheless a double-spanning beam with identical spans and loads could, by virtue of symmetry, be treated as warping-fixed at the central support. Cases 8 and 9 cater for this rare possibility.

Case 10 covers distributed torque varying linearly from zero at one end to $2T/L$ at the other. Results from this and Case 4 can be superimposed to cater for a *Slimflor®* panel in which the beams are non-parallel.

In theory there are many more cases that could have been included. Also, warping fixity is all or nothing; there is no provision for flexible restraint. However, experience suggests that the cases included can answer most practical demands.

Note that the end conditions for torsion are not the same as end conditions for vertical or lateral restraint; it is possible to have one form of restraint without the other - for example an unpropped cantilever can still be restrained against torsion at the free end.

Cautionary notes on use of the formulae

It should always be recognised that the value of T applicable to the graphs and formulae is the **total** applied torque, not the greatest value of torsional moment in the beam. In a simple beam with a central torque, for example, the maximum torsional moment T_{Ed} is only half the applied torque.

In a typical *Slimflor*® construction stage calculation using Case 4, T is the slab reaction per unit length × eccentricity × length of the beam, end-to-end.

Table C.1 Expressions for ϕ, ϕ', ϕ'' and ϕ'''

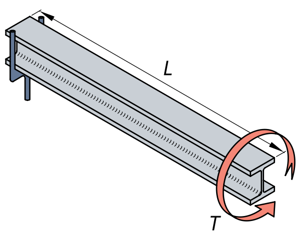
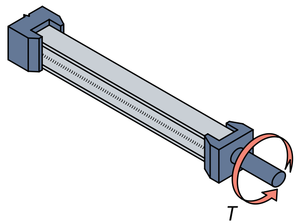
CASE	DESCRIPTION	FORMULA	GRAPH
1 	Torsional restraint at one end only. No end restraint against warping. Point torque T applied at free end. *This case represents the torsional equivalent of a cantilever, but note that the free end may or may not be supported; torsional behaviour is no different.*	$\phi = Tx/(GI_T)$ $\phi' = T/(GI_T)$ $\phi'' = 0$ $\phi''' = 0$ For uniformly distributed torque, ϕ' is not constant and there will be warping. However it is convenient (and safe) to assume a linear variation in ϕ' and end rotation of $TL/(2GI_T)$	Not required for this simple case
2 	Both ends restrained against warping. Equal and opposite torques T applied at the ends.	$\phi = [Ta/(GI_T)]\ \{\tanh[L/(2a)]\cosh(x/a) - \tanh[L/(2a)] + x/a - \sinh(x/a)\}$ $\phi' = [T/(GI_T)]\ \{\tanh[L/(2a)]\sinh(x/a) + 1 - \cosh(x/a)\}$ $\phi'' = [T/(GI_T a)]\ \{\tanh[L/(2a)]\cosh(x/a) - \sinh(x/a)\}$ $\phi''' = [T/(GI_T a^2)]\ \{\tanh[L/(2a)]\sinh(x/a) - \cosh(x/a)\}$	Not provided

Table C.1 (continued)

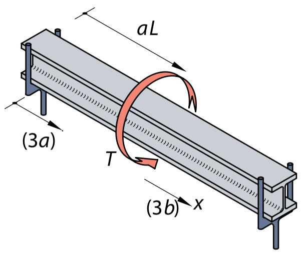
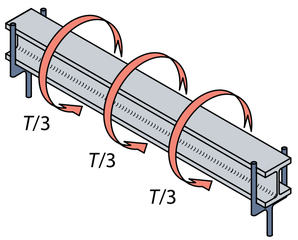
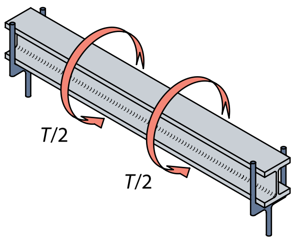
CASE	DESCRIPTION	FORMULA	GRAPH
3 	Torsional restraint at both ends. No end restraint against warping. Point torque T applied at an intermediate point αL from one end of a member of length L.	(a) for $x \le \alpha L$ $\phi = [Ta/(GI_T)]\ \{(1-\alpha)x/a + [\sinh(\alpha L/a)/\tanh(L/a) - \cosh(\alpha L/a)]\sinh(x/a)\}$ $\phi' = [T/(GI_T)]\ \{(1-\alpha) + [\sinh(\alpha L/a)/\tanh(L/a) - \cosh(\alpha L/a)]\cosh(x/a)\}$ $\phi'' = [T/(GI_T a)]\ [\sinh(\alpha L/a)/\tanh(L/a) - \cosh(\alpha L/a)]\sinh(x/a)$ $\phi''' = [T/(GI_T a^2)]\ [\sinh(\alpha L/a)/\tanh(L/a) - \cosh(\alpha L/a)]\cosh(x/a)$ (b) for $x \ge \alpha L$ $\phi = [Ta/(GI_T)]\ [(L-x)\alpha/a + \sinh(\alpha L/a)\sinh(x/a)/\tanh(L/a) - \sinh(\alpha L/a)\cosh(x/a)]$ $\phi' = [T/(GI_T)]\ [-\alpha + \sinh(\alpha L/a)\cosh(x/a)/\tanh(L/a) - \sinh(\alpha L/a)\sinh(x/a)]$ $\phi'' = [T/(GI_T a)]\ [\sinh(\alpha L/a)\sinh(x/a)/\tanh(L/a) - \sinh(\alpha L/a)\cosh(x/a)]$ $\phi''' = [T/(GI_T a^2)]\ [\sinh(\alpha L/a)\cosh(x/a)/\tanh(L/a) - \sinh(\alpha L/a)\sinh(x/a)]$ (c) for special case of $x = \alpha = L/2$ $\phi = [Ta/(GI_T)]\ [L/(4a) + \{\sinh[L/(2a)]/\tanh(L/a) - \cosh(L/(2a)\}\sinh(L/(2a)]$ $\phi'' = [T/(GI_T a)]\ \{\sinh[L/(2a)]/\tanh(L/a) - \cosh[L/(2a)]\}\ \sinh[L/(2a)]$ For quarter and third point loading, use formulae for single point load and add together the results for the two or three applied torques (evaluated for $T/2$ or $T/3$, as appropriate).	Use graphs A (for ϕ) B (for ϕ'')
	Alternatively: Point torque $T/3$ applied at quarter points		
	Or: Point torque $T/2$ applied at third points.		

Table C.1 (continued)

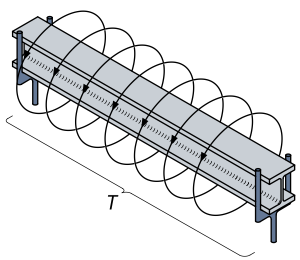
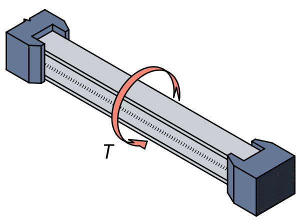
CASE	DESCRIPTION	FORMULA	GRAPH
4 	Torsional restraint at both ends. No end restraint against warping. Uniformly distributed torque T along full length of member.	$\phi = [Ta^2/(GI_T L)]\ \{(xL - x^2)/(2a^2) + \cosh(x/a) - \tanh[L/(2a)]\sinh(x/a) - 1\}$ $\phi' = [Ta/(GI_T L)]\ \{L/(2a) - x/a + \sinh(x/a) - \tanh[L/(2a)]\cosh(x/a)\}$ $\phi'' = [T/(GI_T L)]\ \{-1 + \cosh(x/a) - \tanh[L/(2a)]\sinh(x/a)\}$ $\phi''' = [T/(GI_T La)]\ \{\sinh(x/a) - \tanh[L/(2a)]\cosh(x/a)\}$	Use graphs C (for ϕ) D (for ϕ'')
5 	As Case 3 but fully restrained against both warping and torsion at both ends. *Note: expressions for K_1 etc are given at the foot of this table.*	(a) for $x \le \alpha L$ $\phi = [Ta/(GI_T)]\ \{(K_1 - K_3 + K_4)[\cosh(x/a) - 1] - \sinh(x/a) + x/a\}/(K_1 + 1)$ $\phi' = [T/(GI_T)]\ \{(K_1 - K_3 + K_4)\sinh(x/a) - \cosh(x/a) + 1\}/(K_1 + 1)$ $\phi'' = [T/(GI_T a)]\ \{(K_1 - K_3 + K_4)\cosh(x/a) - \sinh(x/a)\}/(K_1 + 1)$ $\phi''' = [T/(GI_T a^2)]\ \{(K_1 - K_3 + K_4)\sinh(x/a) - \cosh(x/a)\}/(K_1 + 1)$ (b) for $x \ge \alpha L$ $\phi = [Ta/(GI_T)]\ [K_2 + K_5\cosh(x/a) + \{[\cosh(\alpha L/a) - 1]/K_1 + \cosh(\alpha L/a)\}\sinh(x/a) - x/a]/(1 + 1/K_1)$ $\phi' = [T/(GI_T)]\ [K_5\sinh(x/a) + \{[\cosh(\alpha L/a) - 1]/K_1 + \cosh(\alpha L/a)\}\cosh(x/a) - 1]/(1 + 1/K_1)$ $\phi'' = [T/(GI_T a)]\ [K_5\cosh(x/a) + \{[\cosh(\alpha L/a) - 1]/K_1 + \cosh(\alpha L/a)\}\sinh(x/a)]/(1 + 1/K_1)$ $\phi''' = [T/(GI_T a^2)]\ [K_5\sinh(x/a) + \{[\cosh(\alpha L/a) - 1]/K_1 + \cosh(\alpha L/a)\}\cosh(x/a)]/(1 + 1/K_1)$	Not provided

Table C.1 (continued)

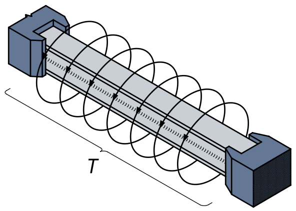
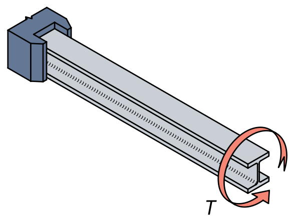
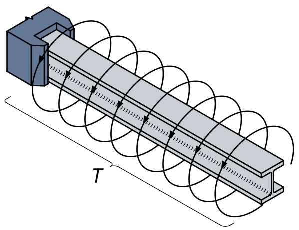
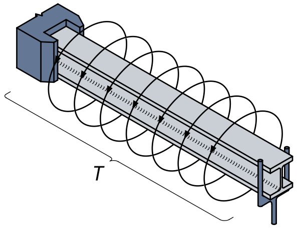
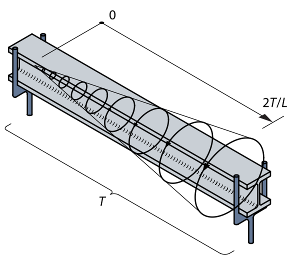
CASE	DESCRIPTION	FORMULA	GRAPH
6 	As Case 4 but fully restrained against both warping and torsion at both ends.	$\phi = [Ta/(2GI_T)]\ \{[1 + \cosh(L/a)][\cosh(x/a) - 1]/\sinh(L/a) + (1 - x/L)x/a - \sinh(x/a)\}$ $\phi' = [T/(2GI_T)]\ \{[1 + \cosh(L/a)]\sinh(x/a)/\sinh(L/a) + 1 - 2x/L - \cosh(x/a)\}$ $\phi'' = [T/(2GI_Ta)]\ \{[1 + \cosh(L/a)]\cosh(x/a)/\sinh(L/a) - 2a/\mathrm{L} - \sinh(x/a)\}$ $\phi''' = [T/(2GI_Ta^2)]\ \{[1 + \cosh(L/a)]\sinh(x/a)/\sinh(L/a) - \cosh(x/a)\}$	Not provided
7 	As Case 1 but one end (at $x = 0$) is restrained against warping as well as against torsion. Point torque T applied at free end. *Note that the free end may or may not be supported; torsional behaviour is no different.*	$\phi = [Ta/(GI_T)]\ \{\tanh(L/a)[\cosh(x/a) - 1] - \sinh(x/a) + x/a\}$ $\phi' = [T/(GI_T)]\ [\tanh(L/a)\sinh(x/a) - \cosh(x/a) + 1]$ $\phi'' = [T/(GI_Ta)]\ [\tanh(L/a)\cosh(x/a) - \sinh(x/a)]$ $\phi''' = [T/(GI_Ta^2)]\ [\tanh(L/a)\sinh(x/a) - \cosh(x/a)]$	Use graph E (for both ϕ and ϕ'')
8 	As Case 1 but one end is restrained against both warping and torsion. Torque T uniformly distributed along the length of the member. *Note that the free end may or may not be supported; torsional behaviour is no different.*	$\phi = [Ta^2/(GI_TL)]\ \{K_8[\cosh(x/a) - 1] - (L/a)\sinh(x/a) + (x/a)[L/a - x/(2a)]\}$ $\phi' = [Ta/(GI_TL)]\ [K_8\sinh(x/a) - (L/a)\cosh(x/a) + (L/a - x/a)]$ $\phi'' = [T/(GI_TL)]\ [K_8\cosh(x/a) - (L/a)\sinh(x/a) - 1]$ $\phi''' = [T/(GI_T\mathrm{a}L)]\ [K_8\sinh(x/a) - (L/a)\cosh(x/a)]$	Use graph E (for both ϕ and ϕ'')

Table C.1 (continued)

CASE	DESCRIPTION	FORMULA	GRAPH
9 	As Case 4 but one end is fully restrained against both warping and torsion.	$\phi = [Ta^2/(GI_T L)]\ \{K_{10}[\tanh(L/a) - x/a - \tanh(L/a)\cosh(x/a) + \sinh(x/a)] + \cosh(x/a)/\cosh(L/a) - 1/\cosh(L/a) - x^2/(2a^2)\}$ $\phi' = [Ta/(GI_T L)]\ \{K_{10}[-1 - \tanh(L/a)\sinh(x/a) + \cosh(x/a)] + \sinh(x/a)/\cosh(L/a) - x/a\}$ $\phi'' = [T/(GI_T L)]\ \{K_{10}[-\tanh(L/a)\cosh(x/a) + \sinh(x/a)] + \cosh(x/a)/\cosh(L/a) - 1\}$ $\phi''' = [T/(GI_T aL)]\ \{K_{10}[-\tanh(L/a)\sinh(x/a) + \cosh(x/a)] + \sinh(x/a)/\cosh(L/a)\}$	Not provided
10 	Torsional restraints at both ends. No end restraint against warping. Torque per unit length varying from zero at LH end ($x = 0$) to $2T/L$ at RH end ($x = L$). Total torque T.	$\phi = [2T/(GI_T)]\ [x/6 - xa^2/L^2 + (a^2/L)\sinh(x/a)/\sinh(L/a) - x^3/(6L^2)]$ $\phi' = [2T/(GI_T)]\ [1/6 - a^2/L^2 + (a/L)\cosh(x/a)/\sinh(L/a) - x^2/(2L^2)]$ $\phi'' = [2T/(GI_T)]\ [(1/L)\sinh(x/a)/\sinh(L/a) - x/L^2]$ $\phi''' = [2T/(GI_T)]\ \{[1/(aL)]\cosh(x/a)/\sinh(L/a) - 1/L^2\}$	Not provided

$K_1 = \{[1 - \cosh(\alpha L/a)]/\tanh(L/a) + [\cosh(\alpha L/a) - 1]/\sinh(L/a) + \sinh(\alpha L/a) - \alpha L/a\}\ /\ \{[\cosh(L/a) + \cosh(\alpha L/a)\cosh(L/a) - \cosh(\alpha L/a) - 1]/\sinh(L/a) + (\alpha - 1)L/a - \sinh(\alpha L/a)\}$

$K_2 = [\cosh(\alpha L/a) - 1]/[K_1 \sinh(L/a)] + [\cosh(\alpha L/a) - \cosh(L/a) + (L/a)\sinh(L/a)]/\sinh(L/a)$

$K_3 = 1/\sinh(L/a) + \sinh(\alpha L/a) - \cosh(\alpha L/a)]/\tanh(L/a)$

$K_4 = \sinh(\alpha L/a) - \cosh(\alpha L/a)]/\tanh(L/a) + 1/\tanh(L/a)$

$K_5 = [1 - \cosh(\alpha L/a)]/[K_1 \tanh(L/a)] + [1 - \cosh(\alpha L/a)\cosh(L/a)]/\sinh(L/a)$

$K_8 = \tanh(L/a)[L/a - \sinh(L/a)] + \cosh(L/a)$

$K_{10} = [L^2/(2a^2) - 1 + 1/\cosh(L/a)]/[\tanh(L/a) - (L/a)]$

Caution: Note the comments in Sections 2.6 and 7.6 before assuming warping fixity at member ends (indicated by the vice-like device).

In every case, T is the total torque acting on the member.

APPENDIX D: SOLUTIONS FOR ϕ AND ITS DERIVATIVES - GRAPHS

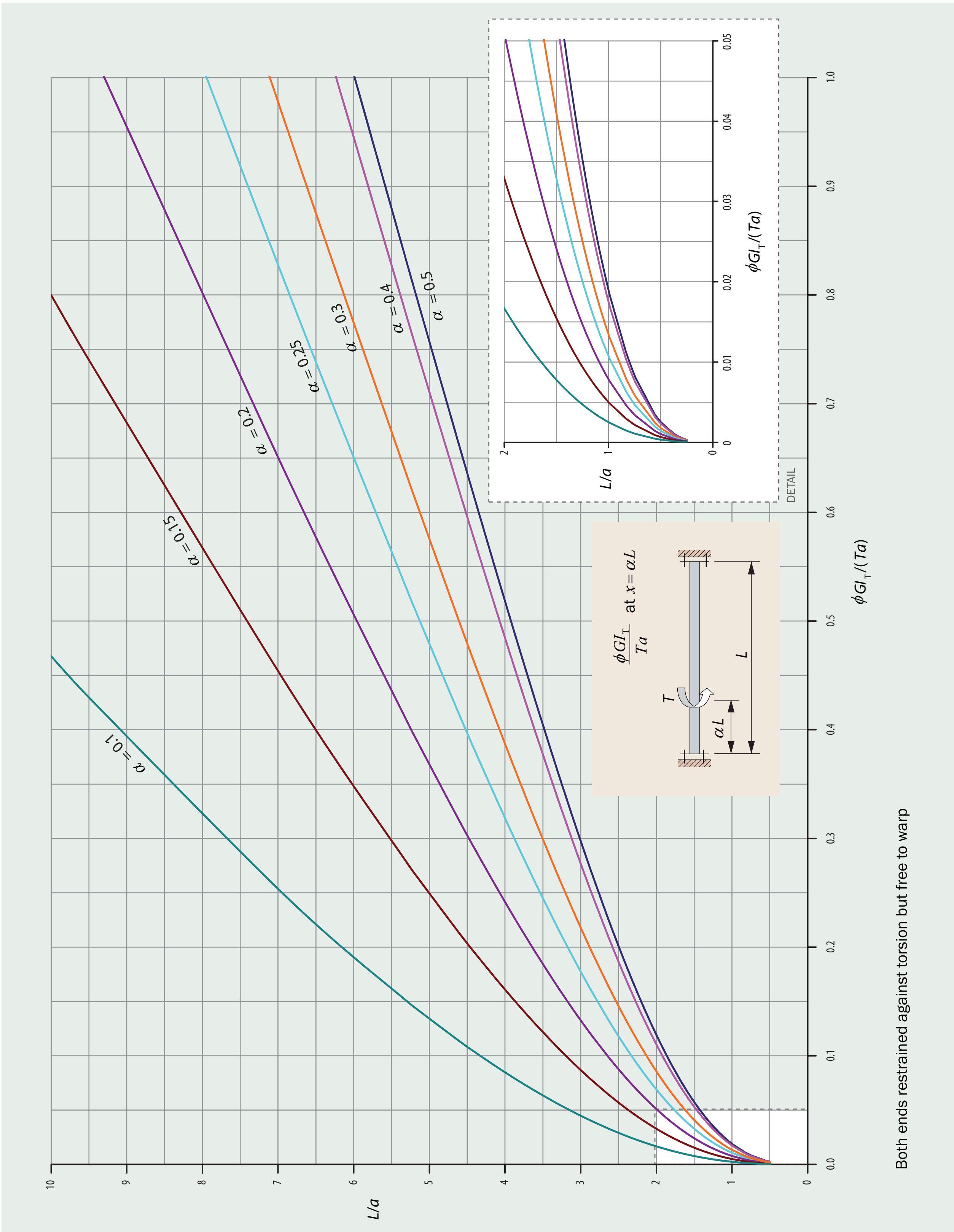
The following graphs are provided:

STRUCTURE	LOADING	GRAPH	PARAMETER
Beam, no warping fixity at ends	Single point torque (Case 3)	A	ϕ
	Single point torque (Case 3)	B	ϕ''
	Point torques at third and quarter points (based on Case 3) UD torque (Case 4)	C	ϕ
	Point torques at third and quarter points (based on Case 3) UD torque (Case 4)	D	ϕ''
Cantilever with warping fixity at support	Point torque at tip (Case 7) UD torque (Case 8)	E	ϕ and ϕ''
Beam, no warping fixity at ends	Uniformly increasing torque (Case 10)	F	ϕ and ϕ''

Cautionary notes on use of the graphs

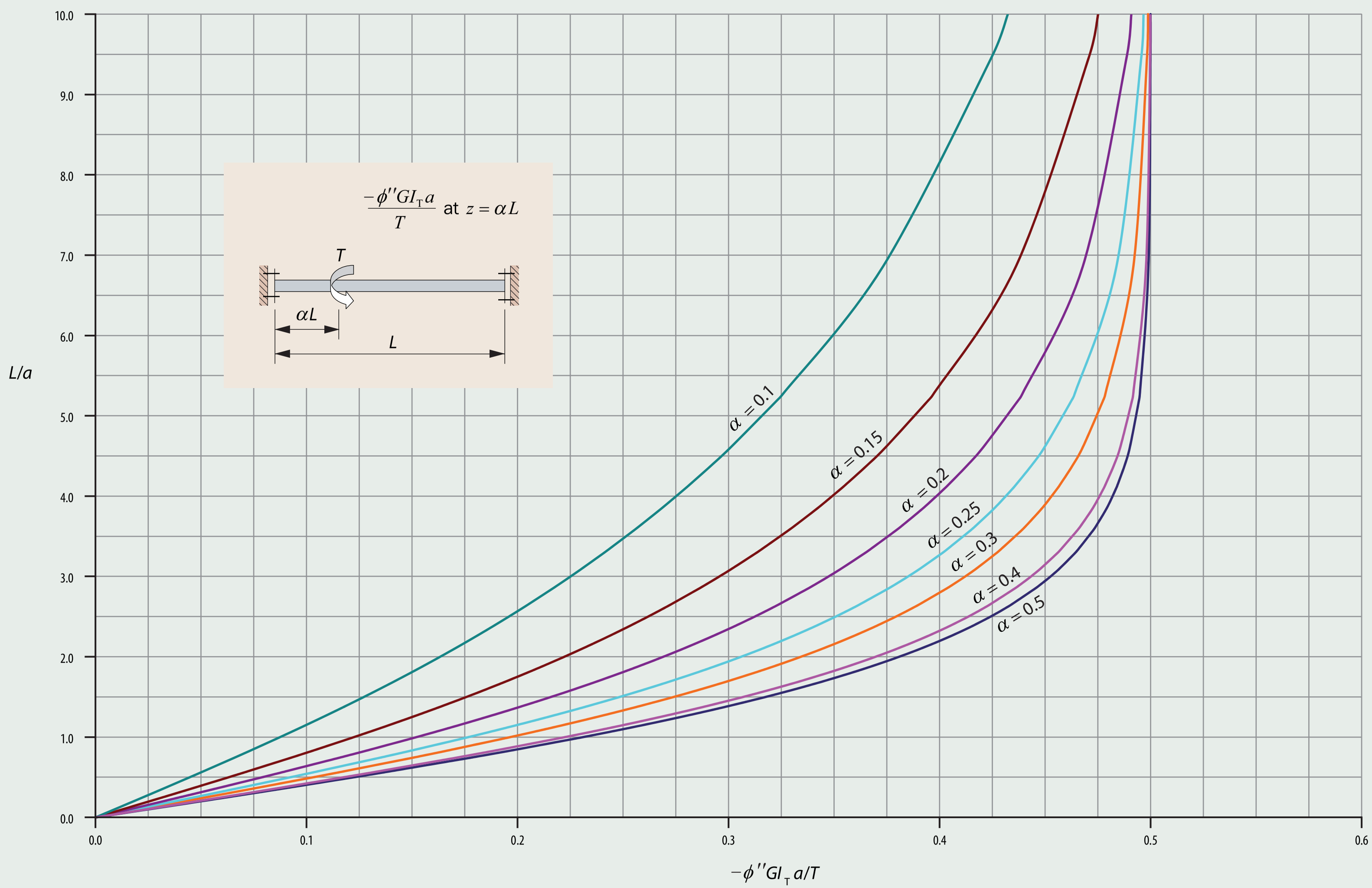
It should always be recognised that the value of T applicable to the graphs and formulae is the **total** applied torque, not the greatest value of torsional moment in the beam. In a simple beam with a central torque, for example, the torsional moment in each half of the beam is only half the applied torque.

In a typical *Slimflor®* construction stage calculation (using graphs C & D), T is the slab reaction per unit length × eccentricity × length of the beam, end-to-end.

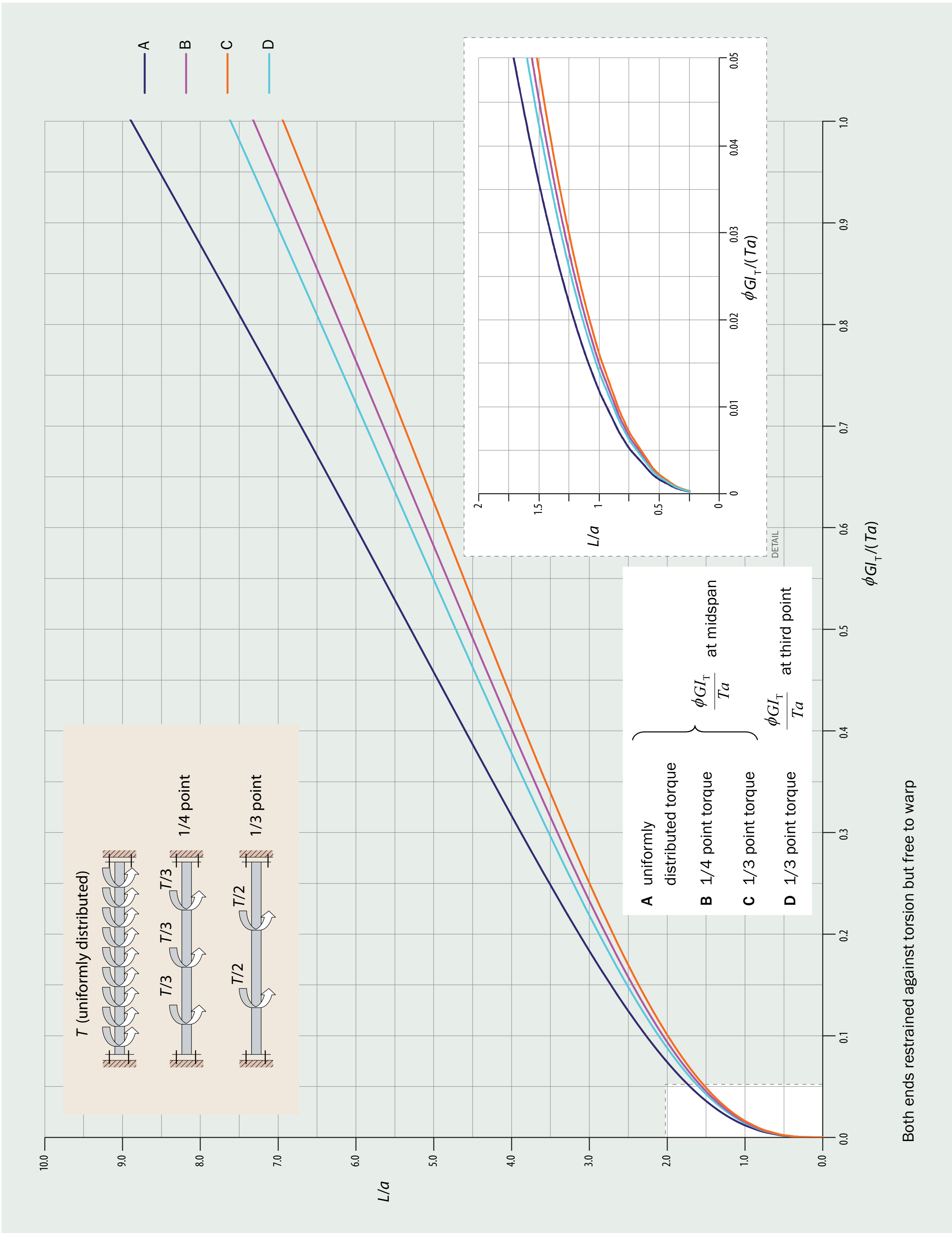


Graph A, for ϕ
Case 3: Single point torque T at $x = \alpha L$

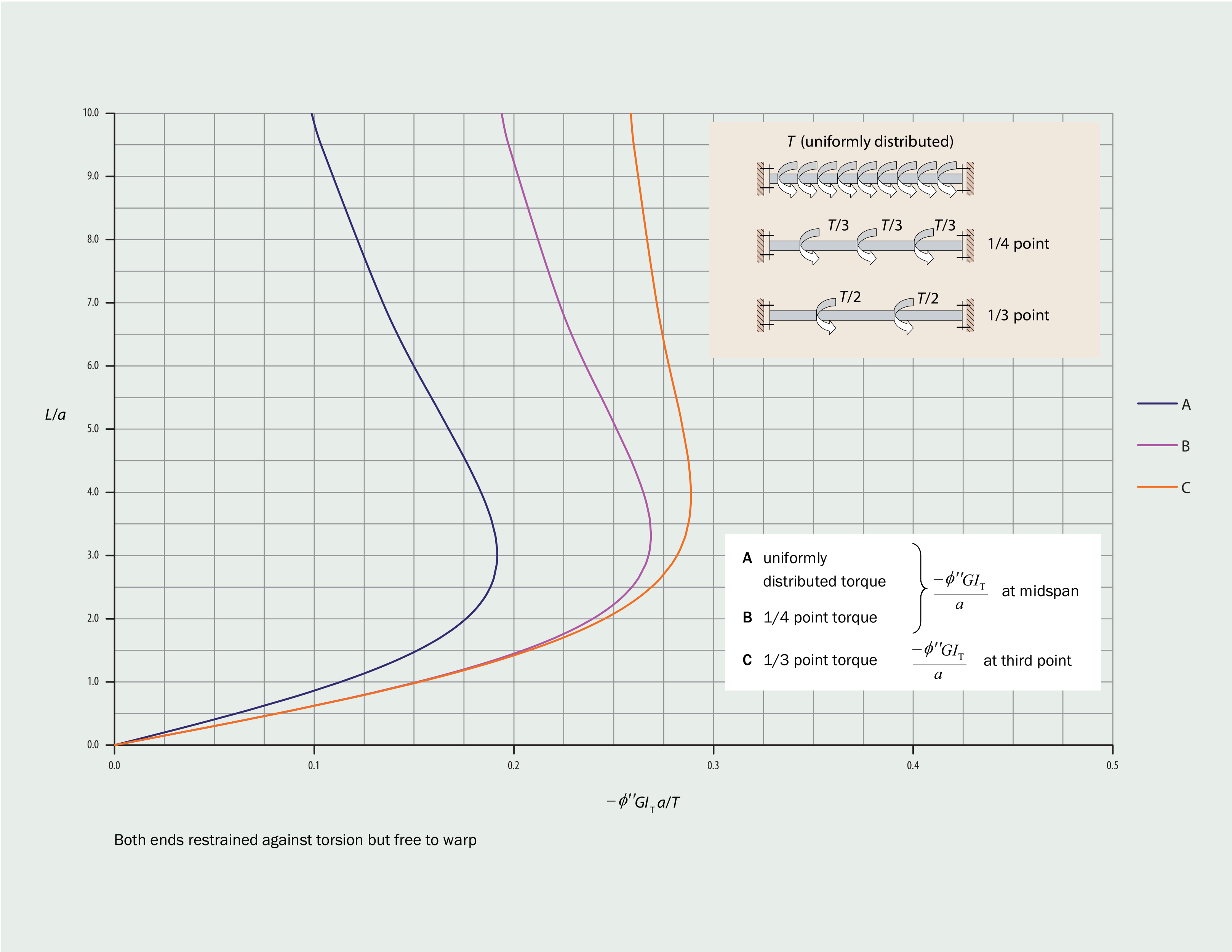
Graph B, for ϕ''
Case 3: Single point torque T at $x = \alpha L$



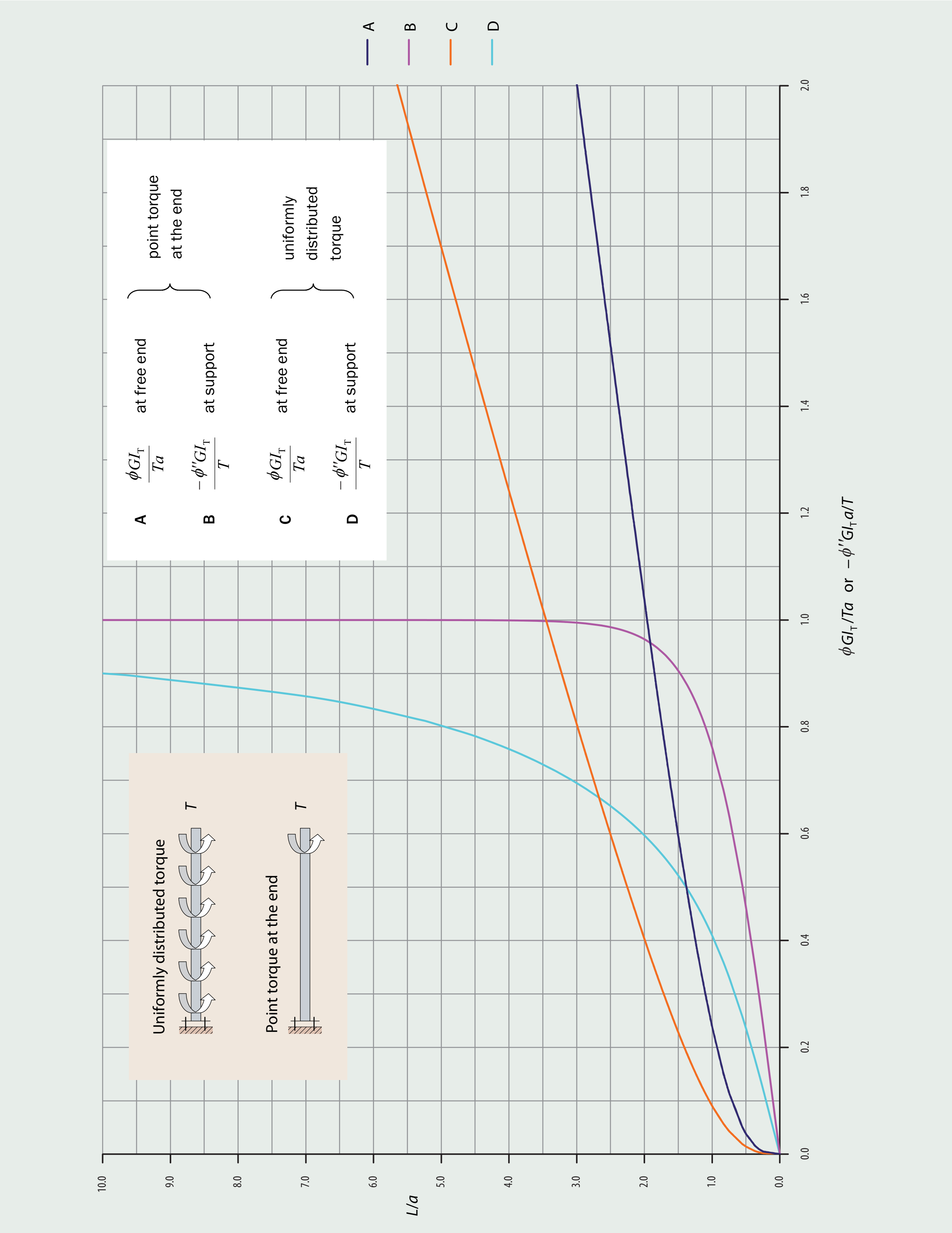
Both ends restrained against torsion but free to warp



Graph C, for ϕ Case 4: Torque T uniformly distributed. Case 3: Symmetrical combinations of third point and quarter point loading

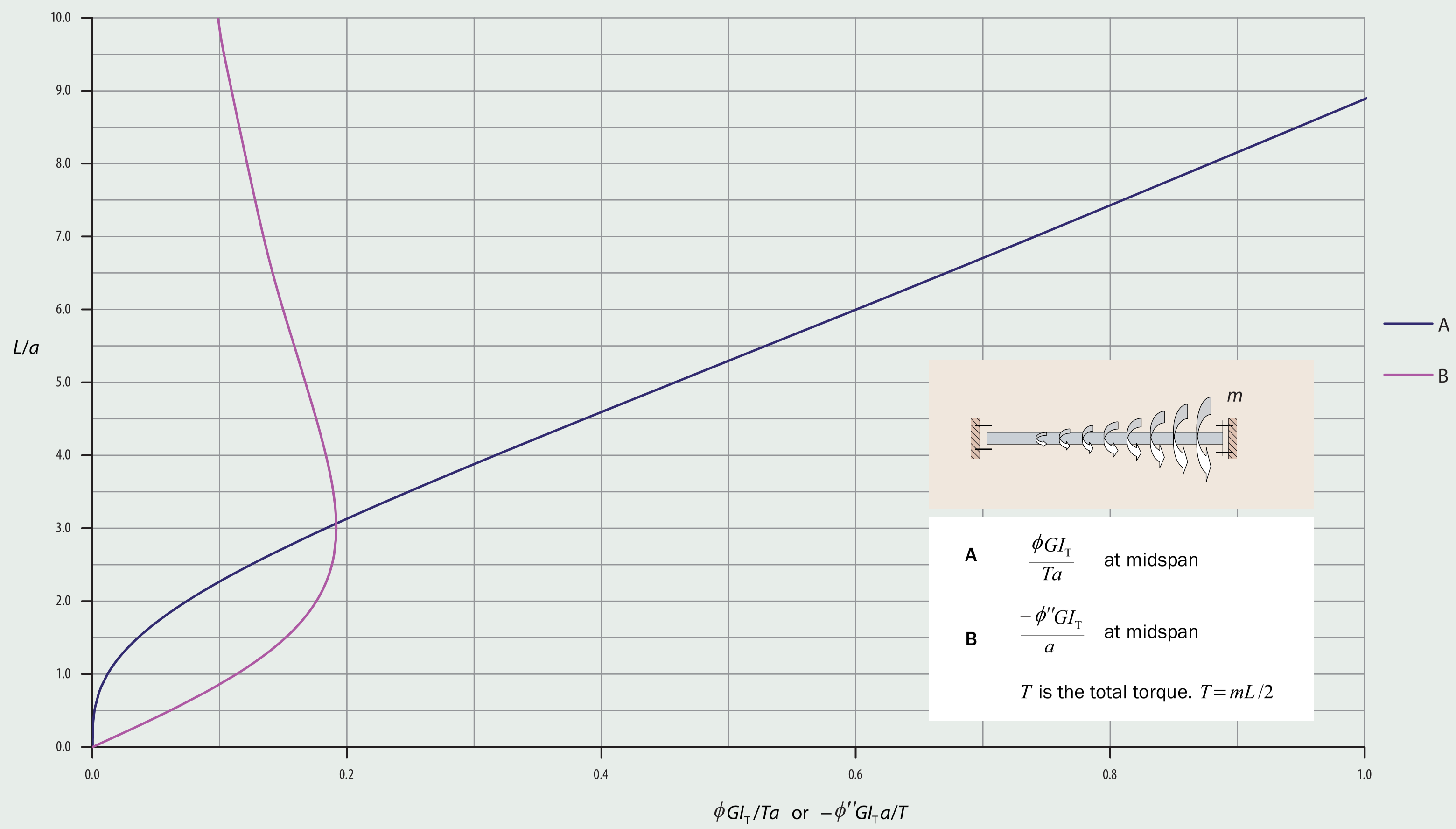


Graph D, for ϕ''
Case 4: Torque T uniformly distributed.
Case 3: Symmetrical combinations of third point and quarter point loading



Graph E, for ϕ and ϕ'' Cantilever with warping fixity at support. Case 7: Point torque at tip. Case 8: Uniformly distributed torque

Graph F, for ϕ and ϕ''
Case 10: Uniformly increasing torque



Both ends restrained against torsion but free to warp

APPENDIX E: DESIGN EXAMPLES

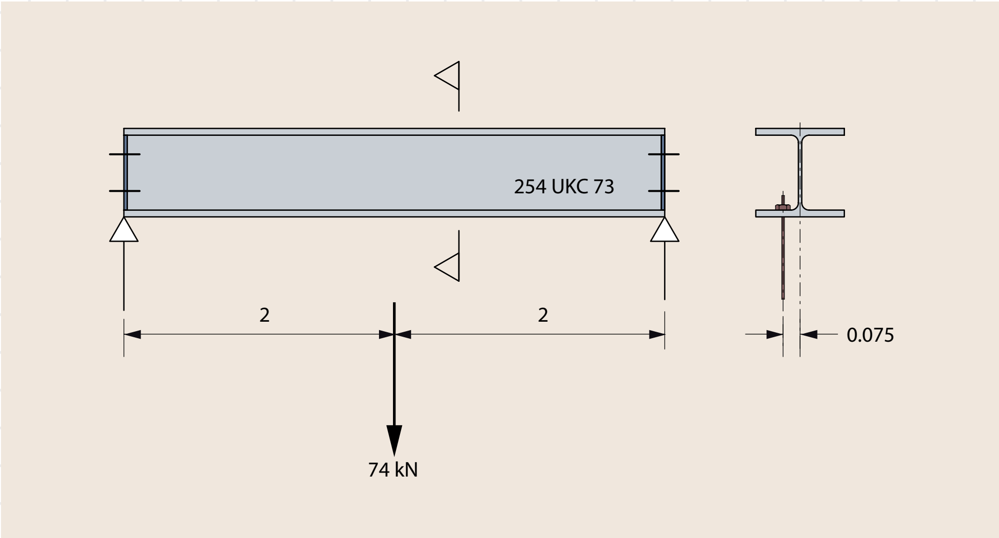
This Appendix presents six design examples, as follows:

Example 1 - Unrestrained beam with eccentric point load

1.1 Configuration

A simply supported beam spans 4 m without intermediate restraint. It is subject to a permanent concentrated load of 74 kN at mid-span, which is attached to the bottom flange at an eccentricity of 75 mm.

Verify the trial section 254UKC73 (S275).



Any restraint provided by the end plate connections against warping is partial, unreliable and unquantifiable. The ends of the member will therefore be assumed to be free to warp.

Note: This example is similar to Example 6 in SCI publication P364. That example only uses a simplified assessment of torsional effects.

1.2 Section properties

For 254 UKC 73 in S275

SCI P363 $b = 254.6$ mm

$h = 254.1$ mm

$t_w = 8.6$ mm

$t_f = 14.2$ mm

$I_w = 0.562$ dm^6

$I_T = 57.6$ cm^4

$r = 12.7$ mm

$A = 9310$ mm^2

$W_{pl,y} = 992$ cm^2

$W_{pl,z} = 465$ cm^2

$I_z = 3910$ cm^4

Table A.2 $a = 1.59$ m

For $t < 16$ mm and S275

BS EN 10025-2, Table 7

$f_y = R_{eH} = 275\ \text{N/mm}^2$

1.3 Actions

1.3.1 Partial factor for actions

BS EN 1990 Table NA.A1.2(B)

Permanent actions $\gamma_G = 1.35$

1.3.2 Combination of actions at ULS

BS EN 1990 presents two options for determining the effect due to combination of actions to be used for the ultimate limit state verification. Here the less favourable combination from Expression (6.10a) and (6.10b) is determined.

Expression (6.10b) will usually be the governing case in the UK, except for cases where the permanent actions are greater than 4.5 times the variable actions.

BS EN 1990 6.4.3.2, NA.2.2.3.2

However, as there are only permanent actions present, 6.10a will be more onerous than 6.10b and so governs the design.

For the concentrated load acting at mid-span with an eccentricity of 75 mm.

$$F_d = \gamma_G G_k = 1.35 \times 74 = 100\ \text{kN}$$

$$T_d = 0.075 F_d = 0.075 \times 100 = 7.5\ \text{kNm}$$

For the beam self weight:

$$f_d = \gamma_G g_k = 1.35 \times 0.716 = 0.97\ \text{kN/m}$$

1.4 Design value of bending moment and shear force

At mid-span:

$$M_{y,Ed} = 100 \times 4/4 + 0.97 \times 4^2/8 = 102\ \text{kNm}$$

At the support:

$$V_{Ed} = 100/2 + 0.97 \times 4/2 = 52\ \text{kN}$$

1.5 Design values of torsional effects at ULS

1.5.1 Simplified assessment of effects

3-1-1/§6.2.7(7)

As a simplification, for members with open cross sections, the effects of St Venant torsion may be neglected.

The torque should then be considered as a couple, applied to the flanges, where the force is given by:

$$F_{w,d} = T_d/(h - t_f) = 7.5/(0.254 - 0.014) = 31.3 \text{ kN}$$

The bending moment in a flange at mid-span is thus:

$$M_{w,Ed} = (31.3 \times 4)/4 = 31.3 \text{ kNm}$$

1.5.2 Assessment of effects, allowing for elastic interaction between St Venant torsion and warping torsion

The flanges are unrestrained against warping at their ends and the beam is subject to a point torque; graphs A and B in Appendix D and the expressions in Case 3 in Appendix C are thus applicable.

For this beam:

$$\frac{L}{a} = \frac{4}{1.59} = 2.52$$

In this case, the torque acts at mid-span and thus $\alpha = 0.5$

Rotation

From Graph A, for $\alpha = 0.5$ and $L/a = 2.52$:

$$\frac{\phi G I_T}{T_d a} = 0.21$$

Therefore

$$\phi = \frac{0.21 \times 7.5 \times 1.59}{81 \times 10^6 \times 57.6 \times 10^{-8}} = 0.053 \text{ rad } (3^\circ)$$

Minor axis moment induced by rotation:

$$M_{z,Ed} = \phi M_{y,Ed} = 0.053 \times 102 = 5.4 \text{ kNm}$$

Warping moment

From Graph B, for $\alpha = 0.5$ and $L/a = 2.52$:

$$-\frac{\phi'' G I_T a}{T} = 0.425$$

$$-\phi'' = \frac{0.425 \times 7.5}{81 \times 10^6 \times 57.6 \times 10^{-8} \times 1.59} = 0.043 \text{ rad/m}^2$$

The warping moment at mid-span is thus:

$$M_{w,Ed} = -EI_f \left(\frac{h - t_f}{2} \right) \phi''$$

where $I_f \approx I_z/2$

Therefore:

$$M_{w,Ed} = \frac{210 \times 10^6 \times 19.55 \times 10^{-6} \times (0.254 - 0.014) \times 0.043}{2} = 21.1 \text{ kNm}$$

This moment occurs in each flange (in opposite directions).

Note: Only in the top flange does warping moment act in the same direction as the minor axis moment induced by rotation.

Commentary: Evaluation of the expressions in Appendix C, Case 3, along the length of the beam would give the following bending moment diagram:

Warping moment due to point torque

It may be noted that the interaction with St Venant torsion has reduced the peak warping moment (relative to that in the simplified assessment, by approximately 33% (21.1 kNm compared to 31.3 kNm).

St Venant torsional moment

The St Venant torsional moment is given by $T = \phi' GI_T$ (see Section 2.3).

No graphs are provided in Appendix D for ϕ', but its value may be obtained from expressions in Appendix C.

For $\alpha = 0.5$ the expression for a point torque simplifies to:

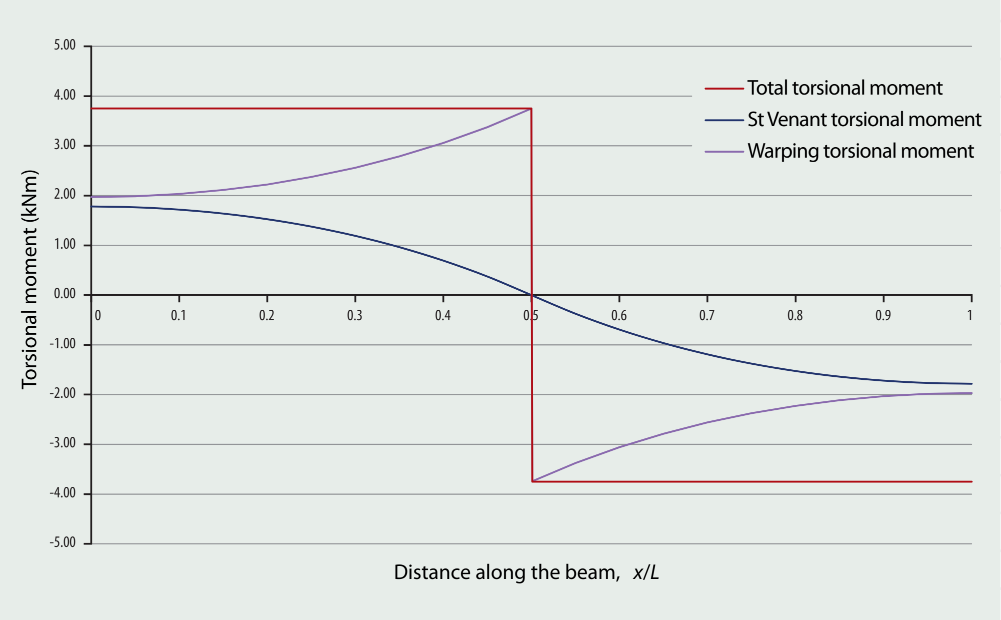
$$\phi' = \frac{T_d}{GI_T} \left\{ \frac{1}{2} + \left[\frac{\sinh(L/2a)}{\tanh(L/a)} - \cosh(L/2a) \right] \cosh(x/a) \right\}$$

At the member end, $x = 0$ and

$$\phi' = \frac{T_d}{GI_T}\left\{\frac{1}{2}+\left[\frac{\sinh 1.26}{\tanh 2.52}-\cosh 1.26\right]\cosh 0\right\} = \frac{T_d}{GI_T}\times 0.238$$

$$T_{t,Ed} = GI_T\phi' = T_d \times 0.238 = 0.238 \times 7.50 = 1.78 \text{ kNm}$$

Further evaluation of the expressions in Appendix C would reveal the sharing of torsional moment between St Venant torsion and warping torsion along the length of the beam. The following plot shows the variation.



Variation of torsional moment along the beam

1.6 Cross sectional resistance

1.6.1 *Bending resistance*

For this Class 1 section, the bending resistance about the major axis is:

3-1-1/§6.2.5(2)

$$M_{y,Rd} = \frac{W_{pl,y}\,f_y}{\gamma_{M0}}$$

$\gamma_{M0} = 1.0$, according to the UK NA

$$M_{y,Rd} = \frac{992\times10^{-6}\times275\times10^{3}}{1.0} = 273 \text{ kNm}$$

$$M_{y,Ed} = 102 \text{ kNm} < M_{y,Rd} = 273 \text{ kNm}$$

The bending resistance about the minor axis is:

$$M_{z,Rd} = M_{pl,z,Rd} = \frac{W_{pl,z}f_y}{\gamma_{M0}} = \frac{465\times10^{-6}\times275\times10^{3}}{1.0} = 128 \text{ kNm}$$

$M_{z,Ed} = 5.4$ kNm $< M_{y,Rd} = 128$ kNm

The warping resistance of one flange is:

$M_{w,Rd} = M_{pl,f,Rd} = M_{pl,z,Ed}/2 = 64$ kNm $> M_{w,Ed} = 21.1$ kNm

Consider the plastic interaction criterion, as given in Section 3.1.2:

$$\left[\frac{M_{y,Ed}}{M_{pl,y,Rd}}\right]^2 + \frac{M_{w,Ed}}{M_{pl,f,Rd}} + \frac{M_{z,Ed}}{M_{pl,z,Rd}} \leq 1$$

$$\left[\frac{102}{273}\right]^2 + \frac{21.1}{64} + \frac{5.4}{128} = 0.14 + 0.33 + 0.04 = 0.51 \leq 1 \quad \text{OK}$$

In this example, the loading at mid-span is applied through a tension rod that passes through a 27 mm diameter hole in the bottom flange. It is therefore necessary to find out whether the bending resistances need to be reduced to take account of the hole.

According to 6.2.5 (4), fastener holes in a tension flange may be ignored provided that:

$$\frac{A_{f,net}\,0.9 f_u}{\gamma_{M2}} \geq \frac{A_f f_y}{\gamma_{M0}}$$

For S275 steel, $f_u = 430$ N/mm^2

BS EN 10025-2 $\gamma_{M2} = 1.10$, according to the UK NA

$A_f = 254 \times 14.2 = 3610$ mm^2

$A_{f,net} = A_f - 27 \times 14.2 = 3220$ mm^2

$$\frac{3220 \times 0.9 \times 430}{1.1} \times 10^{-3} = 1130 \text{ kN} \quad \text{and} \quad \frac{3610 \times 275}{1.0} \times 10^{-3} = 993 \text{ kN}$$

So, the hole may be ignored for bending about the major axis.

For lateral bending of the bottom flange, there is no simple criterion to permit neglecting the hole. The value of the section modulus for transverse bending should therefore be evaluated for the flange with the hole, although the calculation of an appropriate plastic modulus for a flange with a hole on one side is not straightforward. Considering here that the hole removes approximately 20% of the area of one flange outstand, that the utilization of the top flange calculated above is only 51% and that the warping moment and minor axis moment are in opposite directions in the bottom flange, the adequacy of the bottom flange is judged satisfactory.

1.6.2 Shear resistance

Plastic shear resistance

Without torsion, the plastic shear resistance of the beam is given by:

$$V_{\mathrm{pl,Rd}} = \frac{A_{\mathrm{v}} f_{\mathrm{y}} / \sqrt{3}}{\gamma_{\mathrm{M0}}}$$

For an I section:

$A_{\mathrm{v}} = A - 2bt_{\mathrm{f}} + (t_{\mathrm{w}} + 2r)t_{\mathrm{f}}$, but not less than $\eta h_{\mathrm{w}} t_{\mathrm{w}}$

$= 9310 - 2 \times (254 \times 14.2) + (8.6 + 2 \times 12.7) \times 14.2 = 2560\ \mathrm{mm}^2$

$h_{\mathrm{w}} = h - 2t_{\mathrm{f}} = 254 - (2 \times 14.2) = 226\ \mathrm{mm}$

3-1-5/NA.2.4 $\eta = 1.0$ (according to UK NA to BS EN 1993-1-5)

Thus $\eta h_{\mathrm{w}} t_{\mathrm{w}} = 1.0 \times 226 \times 8.6 = 1940\ \mathrm{mm}^2$

$2560 > 1940$

Therefore $A_{\mathrm{v}} = 2560\ \mathrm{mm}^2$

$$V_{\mathrm{pl,Rd}} = \frac{2560 \times 275 / \sqrt{3}}{1.0} \times 10^{-3} = 406\ \mathrm{kN}$$

Reduced shear resistance in the presence of torsion

The shear resistance is reduced by the presence of St Venant torsional shear stress in the web.

In order to calculate the reduction in available resistance due to torsion, it is necessary to evaluate the torsional shear stress in the web. This is given by:

$\tau_{\mathrm{t,Ed}} = T_{\mathrm{t,Ed}} \times t / I_{\mathrm{T}}$

$= 1.78 \times 10^6 \times 8.6 / 57.6 \times 10^4 = 26.6\ \mathrm{N/mm}^2$

3-1-1/6.2.7(9) The reduction factor $= \sqrt{1 - \dfrac{\tau_{\mathrm{t,Ed}}}{1.25 f_{\mathrm{y}} / \sqrt{3}}} = \sqrt{1 - \dfrac{26.6\sqrt{3}}{1.25 \times 275}} = 0.93$

$V_{\mathrm{pl,T,Rd}} = 0.93 \times 406 = 378\ \mathrm{kN}$

$V_{\mathrm{Ed}} = 52\ \mathrm{kN} < V_{\mathrm{pl,T,Rd}} = 378\ \mathrm{kN}$ OK

Note that, with this level of shear stress there would be no reduction in plastic bending resistance, even if the two effects were coexistent. In this case, there is no St Venant torsional moment at mid-span.

1.7 Buckling resistance

1.7.1 Lateral torsional buckling resistance

The buckling resistance moment is determined from:

3-1-1/§6.3.2.1(1) $$M_{\mathrm{b,Rd}} = \frac{\chi_{\mathrm{LT}} W_{\mathrm{y}} f_{\mathrm{y}}}{\gamma_{\mathrm{M1}}}$$

$\gamma_{M1} = 1.0$, according to the UK NA

For this Class 1 section, $W_y = W_{pl,y}$

The value of the reduction factor χ_{LT} is determined from a buckling curve according to the non dimensional slenderness $\bar{\lambda}_{LT}$, which is given by:

3-1-1/§6.3.2.1(1)

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}}$$

For the configuration of this beam, the elastic critical moment M_{cr} is given by *LTBeam* as $M_{cr} = 1049$ kNm

Therefore, $\bar{\lambda}_{LT} = \sqrt{\frac{273}{1049}} = 0.51$

3-1-1/NA.2.17

For a doubly symmetric rolled section with $h/b < 2$, the UK NA directs that buckling curve b of 6.3.2.3 should be used.

Buckling curve b

For $\bar{\lambda}_{LT} = 0.51$, $\chi_{LT} = 0.95$

3-1-1/§6.3.2.1(1)

$$M_{b,Rd} = \chi_{LT}\frac{W_y f_y}{\gamma_{M1}} = \frac{0.95 \times 992 \times 10^{-6} \times 273 \times 10^3}{1.0} = 259 \text{ kNm}$$

1.7.2 Interaction of LTB with minor axis bending and torsion

As discussed in Section 3.2, use the formula in Annex A of BS EN 1993-6. For $\gamma_{M0} = \gamma_{M1}$ this may be re-expressed as:

$$\frac{M_{y,Ed}}{M_{b,Rd}} + \frac{C_{mz} M_{z,Ed}}{M_{z,Rd}} + \frac{k_w k_{zw} k_\alpha M_{w,Ed}}{M_{w,Rd}} \leq 1$$

3-1-1/Table B.3

Here:

$C_{mz} = 0.9$ (near-triangular bending moment diagram)

$$k_w = 0.7 - 0.2\frac{M_{w,Ed}}{M_{w,Rd}} = 0.7 - 0.2 \times \frac{21.1}{64} = 0.63$$

$$k_{zw} = 1 - \frac{M_{z,Ed}}{M_{z,Rd}} = 1 - \frac{5.4}{128} = 0.96$$

$$k_\alpha = 1/(1 - \frac{M_{y,Ed}}{M_{cr}}) = 1/(1 - \frac{102}{1049}) = 1.11$$

The criterion is evaluated as:

$$\frac{102}{259} + \frac{0.9 \times 5.4}{128} + \frac{0.63 \times 0.96 \times 1.11 \times 21.1}{64} = 0.39 + 0.04 + 0.23 = 0.66 \text{ OK}$$

Therefore, the buckling resistance is satisfactory.

1.8 Serviceability limit state

The partial factor γ_G at SLS is 1.0, compared with its value of 1.35 at ULS. Hence the SLS action effects are as for ULS $\times$ $1/1.35$

$$\phi_{ser} \quad = 0.053/1.35 = 0.039 \text{ rad } (2.25^\circ)$$

There are no commonly agreed limits for permanent deflections at SLS and the matter is for the designer's judgement.

In this example, the designer might need to consider the effect of such a rotation on the suspension rod, unless it is attached in a way which permits rotation. If, instead of a suspension rod, the load were connected via a cable attached to an eye bolt, a rotation in excess of two degrees might be judged less of a concern.

Example 2 - Crane beam subject to two wheel loads

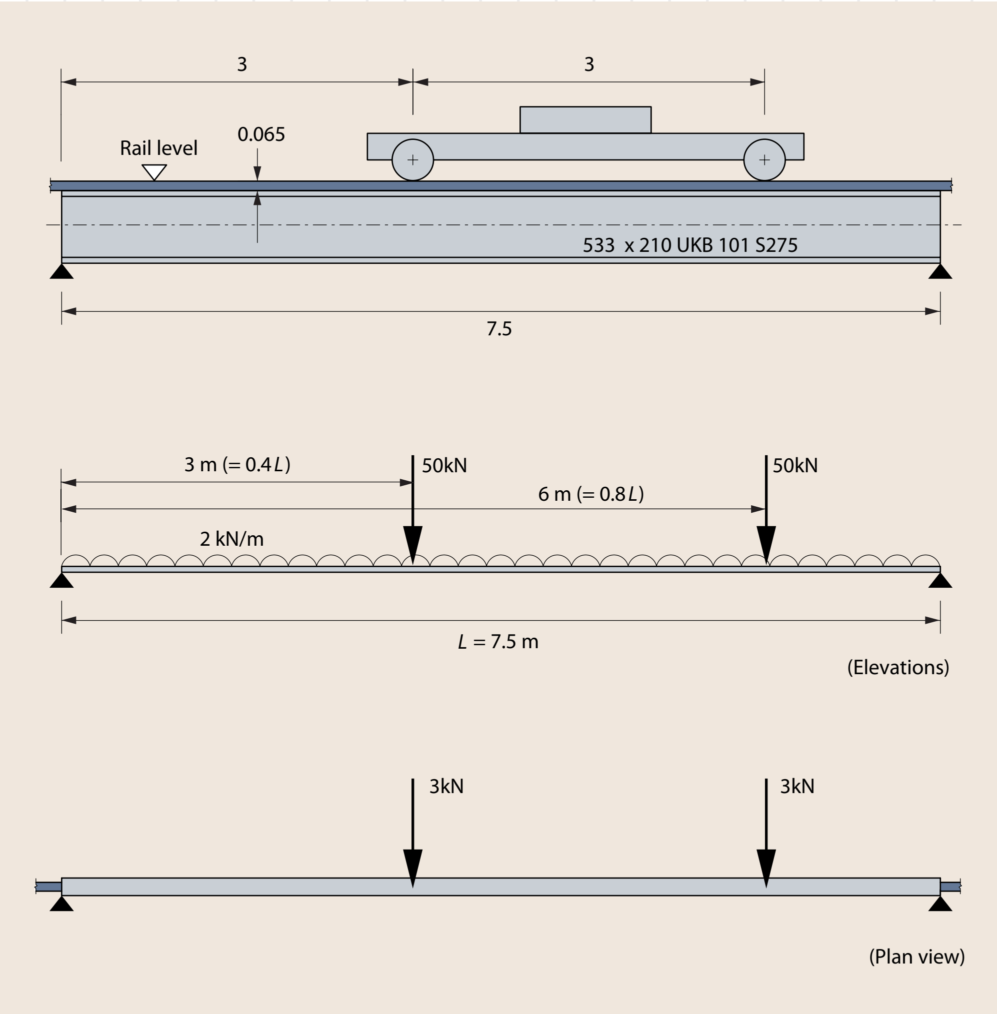
2.1 Configuration

A crane beam spans 7.5 m without intermediate restraint. Verify the chosen 533 × 210 UKB 101 section under the condition shown below, in which two wheel loads 3 m apart act at rail level 65 mm above the beam.

The ULS design values of the loads from the wheels of the crane are 50 kN vertical together with 3 kN horizontal.

Allow 2 kN/m for the design value of the self weight of the beam and crane rail.

Consider the design effects for the location shown below (which gives maximum vertical bending moment).



Assume that an elastomeric pad will be provided between the rail and the beam. According to EN 1993-6, 6.3.2.2(2), the vertical wheel reaction should then be taken as being effectively applied at the level of the top of the flange and the horizontal load at the level of the rail.

2.2 Section properties

SCI P363

For 533 × 210 UKB101, section properties include:

h = 536.7 mm

b = 210 mm

t_w = 10.8 mm

t_f = 17.4 mm

$W_{pl,y}$ = 2610 cm^3

$W_{pl,z}$ = 399 cm^3

r = 12.7 mm

A = 12900 mm^2

I_z = 26.8 cm^4 (= 26.8 × 10^{-8} m^4)

I_T = 101 cm^4 (= 101 × 10^{-8} m^4)

I_w = 1.8 dm^6 (= 1.81 × 10^{-6} m^6)

Table A.1

a = 2.16 m

I_f = $I_z/2$ = 13.4 cm^4 (13.4 × 10^{-6} m^4)

BS EN 10025-2, Table 7

For $t > 16$ mm and S275

f_y = R_{eH} = 265 N/mm^2

2.3 Design values of vertical and horizontal bending moments and shear

The bending moment due to the vertical point loads and the self weight UDL is:

$M_{y,Ed}$ = (50 × 0.6 + 50 × 0.2) × 3.0 + 7.5 × 3.0 – 2 × 3.0^2/2 = 133.5 kNm

The bending moment due to the horizontal point loads is:

$M_{z,Ed}$ = (3 × 0.6 + 3 × 0.2) × 3.0 = 7.2 kNm

Note: This is not the full $M_{z,Ed}$ *since rotation will induce an additional* ϕM_y.

At the left hand support:

V_{Ed} = 50 (0.6 + 0.2) = 40 kN under the given loading

V_{Ed} = 50 (1 + 0.6) = 80 kN with the left hand wheel adjacent to the support

2.4 Design values of torsional effects at ULS

The member is subject to horizontal forces at two points, applied at rail level.

Distance from centroid/shear centre of beam:

d = 536.7/2 + 65 = 333 mm

Torque applied at each position = $333 \times 3 \times 10^{-3} = 1.0$ kNm

2.4.1 Simplified assessment of effects

3-1-1/§6.2.7(7)

As a simplification, for members with open cross sections, the effects of St Venant torsion may be neglected.

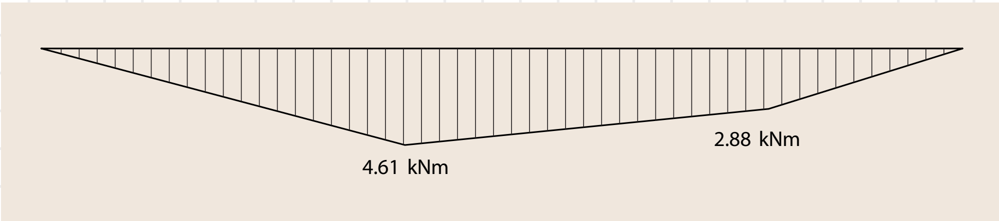
Each torque should then be considered as a couple, applied to the flanges, where the force is given by:

$$F_{w,d} = T_d/(h - t_f) = 1.00 / (0.537 - 0.017) = 1.92 \text{ kN}$$

The bending moments in a flange are thus:

$$M_{w,Ed} = (1.92 \times 0.6 + 1.92 \times 0.2) \times 3.0 = 4.61 \text{ kNm} \quad \text{at the left wheel}$$

$$M_{w,Ed} = (1.92 \times 0.8 + 1.92 \times 0.4) \times 1.5 = 2.88 \text{ kNm} \quad \text{at the right wheel}$$



Bending moment diagram for warping moment (simplified assessment)

2.4.2 Assessment of effects, allowing for elastic interaction between St Venant torsion and warping torsion

The flanges are assumed to be unrestrained against warping at the member ends and thus Graphs A and B in Appendix D and the expressions in Case 3 in Appendix C are applicable. Since the graphs do not cater for $\alpha > 0.5$, the expressions in the Appendix will be used directly.

For this beam:

$$\frac{L}{a} = \frac{7.5}{2.16} = 3.47$$

Concentrated torques $T_d = 1$ kNm at $\alpha = 0.4$ and 0.8 will be considered in turn. Since the calculation is a linear elastic one, the principle of superposition applies.

Rotations

Appendix C, Case 3

The rotations in the beam ϕ are given by:

$$\phi = \frac{T_d}{GI_T}\left\{(1-\alpha)\frac{x}{a} + \left[\frac{\sinh\frac{\alpha L}{a}}{\tanh\frac{L}{a}} - \cosh\frac{\alpha L}{a}\right]\sinh\frac{x}{a}\right\} \quad \text{for } x \le \alpha L$$

$$\phi = \frac{T_d}{GI_T}\left[(L-x)\frac{\alpha}{a} + \frac{\sinh\frac{\alpha L}{a}}{\tanh\frac{L}{a}}\sinh\frac{x}{a} - \sinh\frac{\alpha L}{a}\cosh\frac{x}{a}\right] \quad \text{for } x \geq \alpha L$$

At the left-hand wheel, due to its torque ($\alpha = 0.4$ and $x = \alpha L$):

$$\phi = \frac{T_d a}{GI_T}\left[\frac{0.6\times 3.0}{2.16} + \frac{\sinh(3.0/2.16)}{\tanh(7.5/2.16)}\sinh(3.0/2.16) - \sinh(3.0/2.16)\cosh(3.0/2.16)\right]$$

$$= \frac{T_d a}{GI_T}[0.371]$$

At the left-hand wheel due to the right-hand wheel torque ($\alpha = 0.8$ and $x < \alpha L$)

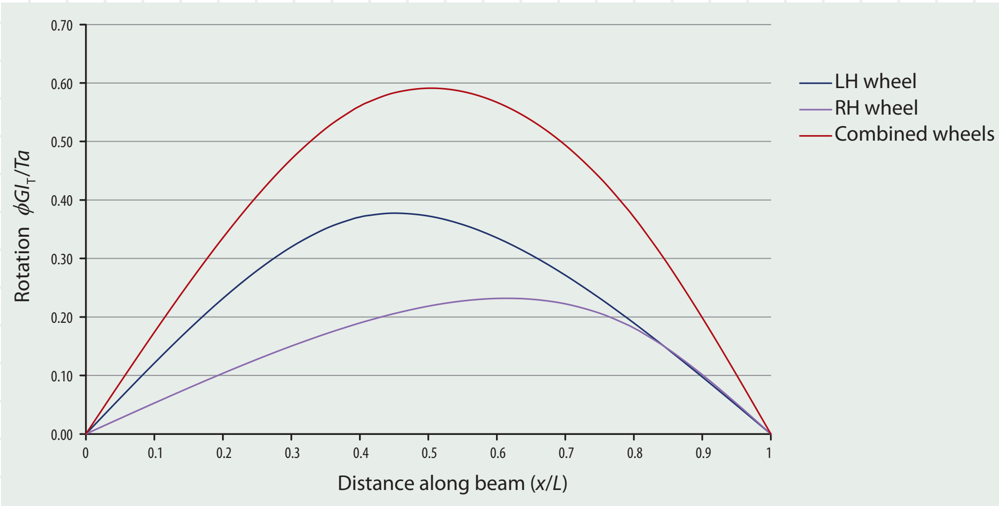
$$\phi = \frac{T_d a}{GI_T}\left[\frac{0.2\times 3.0}{2.16} + \left[\frac{\sinh(6.0/2.16)}{\tanh(7.5/2.16)} - \cosh(6.0/2.16)\right]\sinh(3.0/2.16)\right]$$

$$= \frac{T_d a}{GI_T}[0.190]$$

The torque at each location is 1.00 kNm, hence superposition gives:

$$\phi = \frac{T_d a}{GI_T}[0.371+0.190] = \frac{1.00\times 2.16}{81\times 10^6\times 101\times 10^{-8}}\times 0.561 = 0.0146 \text{ rad } (0.84^\circ)$$

Commentary: Evaluating the rotation along the beam would give the following deflection diagram:



Rotation of crane beam due to two pointloads at 0.4L and 0.8L

The maximum rotation occurs between the two wheels and is slightly greater than that at the left-hand wheel.

Minor axis bending

The additional minor axis bending moment due to the rotation is:

$$M_{z,Ed} = 0.0146\,M_{y,Ed} = 0.0146 \times 133.5 = 1.95 \text{ kNm}$$

The total minor axis moment is thus:

$M_{z,Ed}$ = 1.95 + 7.2 = 9.2 kNm

Warping moment

The warping moment in the flange depends on ϕ'', which is given by:

$$\phi'' = \frac{T_d}{GI_T a}\left\{\left[\frac{\sinh\frac{\alpha L}{a}}{\tanh\frac{L}{a}} - \cosh\frac{\alpha L}{a}\right]\sinh\frac{x}{a}\right\} \quad \text{for } x \le \alpha L$$

$$\phi = \frac{T_d}{GI_T a}\left[\frac{\sinh\frac{\alpha L}{a}}{\tanh\frac{L}{a}}\sinh\frac{x}{a} - \sinh\frac{\alpha L}{a}\cosh\frac{x}{a}\right] \quad \text{for } x \ge \alpha L$$

At the left-hand wheel, due to its torque ($\alpha = 0.4$ and $x = \alpha L$):

$$\phi = \frac{T_d}{GI_T a}\left[\frac{\sinh(3.0/2.16)}{\tanh(7.5/2.16)}\sinh(3.0/2.16) - \sinh(3.0/2.16)\cosh(3.0/2.16)\right]$$

$$= \frac{T_d}{GI_T a}[0.462]$$

At the left-hand wheel due to the right-hand wheel torque ($\alpha = 0.8$ and $x < \alpha L$):

$$\phi = \frac{T_d}{GI_T a}\left[\frac{\sinh(6.0/2.16)}{\tanh(7.5/2.16)} - \cosh(6.0/2.16)\right]\sinh(3.0/2.16)$$

$$= \frac{T_d}{GI_T a}[0.088]$$

Superposition gives:

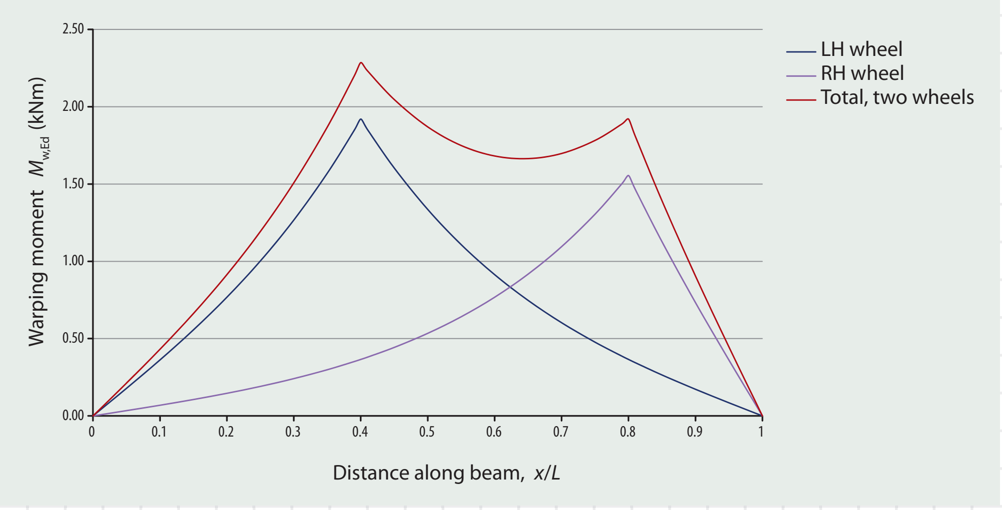
$$\phi'' = \frac{T_d}{GI_T a}[-0.462 - 0.088] = \frac{1.00}{81\times10^6 \times 101\times10^{-8} \times 2.16}\times[-0.550] = -3.11\times10^{-3}$$

The warping moment in each flange is given by:

$$M_{w,Ed} = \pm\frac{EI_w}{(h-t_f)}\phi'' = \frac{210\times10^6\times1.81\times10^{-6}}{(0.537-0.017)}\times3.11\times10^{-3} = \pm 2.28 \text{ kNm}$$

Alternatively, $M_{w,Ed}$ could be derived directly from the value [−0.550] above using $M_{w,Ed} = \pm T_d a/(h - t_f) \times [0.550]$

Commentary: Evaluating the warping moment along the beam would give the following bending moment diagram:

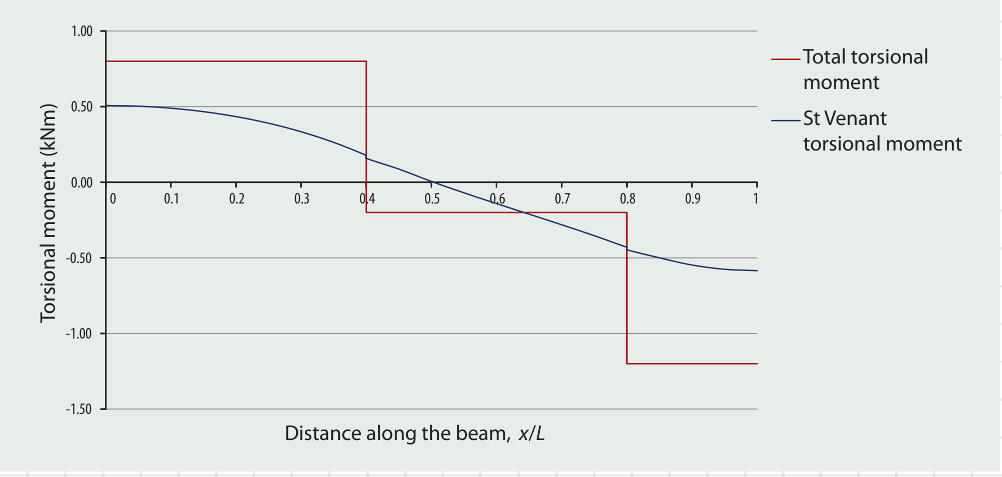


Warping moment due to two point loads

It may be noted that the interaction with St Venant torsion has reduced the peak warping torsional moment, relative to that determined by the simplified assessment in 2.4.1, by approximately 50% (2.29 kNm compared to 4.61 kNm).

There is an equal and opposite warping moment in the bottom flange but only in the top flange does warping moment act in the same direction as M_z.

Further calculations would reveal the sharing of torsional moment between St Venant torsion and warping torsion, along the beam. The following plot shows the variation.



Variation of torsional moment along the beam

2.5 Cross sectional resistance

2.5.1 Bending resistance

For this Class 1 section, the bending resistance about the major axis is:

3-1-1/§6.2.5(2)

$$M_{y,Rd} = \frac{W_{pl,y} f_y}{\gamma_{M0}}$$

γ_{M0} = 1.0, according to the UK NA

$$M_{y,Rd} = \frac{2610 \times 10^{-6} \times 265 \times 10^{3}}{1.0} = 692 \text{ kNm}$$

$M_{y,Ed}$ = 133.5 kNm < $M_{y,Rd}$ = 692 kNm

The bending resistance about the minor axis is:

$$M_{z,Rd} = \frac{W_{pl,z}\, f_y}{\gamma_{M0}} = 399 \times 10^{-6} \times 265 \times 10^3 = 106 \text{ kNm} > M_{z,Ed} = 9.2 \text{ kNm}$$

The bending resistance of one flange is:

$M_{f,Rd} = M_{z,Rd}/2 = 53 \text{ kNm} > M_{w,Ed} = 2.28 \text{ kNm}$

By inspection, the plastic interaction criterion in Section 3.1.2 is satisfied.

2.5.2 Shear resistance

The plastic shear resistance in the absence of torsion is:

3-1-1/§6.2.6(2)

$$V_{pl,Rd} = \frac{A_v V_{Rd} / \sqrt{3}}{\gamma_{M0}}$$

where:

6.2.6(3) $A_v = A - 2b_{tf} + (t_w + 2r)\, t_f$ but not less than $\eta h_w t_w$

$12900 - (2 \times 210 \times 17.4) + (10.8 + 2 \times 12.7) \times 17.4 = 6220 \text{ mm}^2$

$h_w = h - 2t_f = 536.7 - 17.4 = 501.9 \text{ mm}$

NA.2.4 η = 1.0 according to the UK NA to BS EN 1993-1-5

$\eta h_w t_w = 1 \times 501.9 \times 10.8 = 5420 \text{ mm}^2$

$6220 \text{ mm}^2 > 5420 \text{ mm}^2$

Therefore:

$A_v = 6220 \text{ mm}^2$

Therefore:

$$V_{pl,Rd} = \frac{6220 \times \left(265 / \sqrt{3}\right)}{1.0} \times 10^{-3} = 952 \text{ kN}$$

The maximum St Venant torsional moment, at the right hand support, is $T_{t,Ed} = 0.6$ kNm (calculation not given here but may be seen on plot above) hence the St Venant shear stress in the web is:

$\tau_{t,Ed} = T_{t,Ed} \times t_w / I_T$

$= 0.60 \times 10^6 \times 10.8 / (101 \times 10^4) = 6 \text{ N/mm}^2$

The reduction of shear resistance due to St Venant torsion is thus small and by inspection of the values of V_{Ed}, the shear resistance of the beam is adequate.

2.6 Buckling resistance

2.6.1 Lateral torsional buckling resistance

3-1-1/§6.3.2.1(1) $M_{b,Rd} = \dfrac{\chi_{LT} W_y f_y}{\gamma_{M1}}$

For class 1 and class 2 sections $W_y = W_{pl,y}$

The reduction factor for lateral torsional buckling χ_{LT} is determined directly from a buckling curve using $\bar{\lambda}_{LT}$

3-1-1/§6.3.2.1(1) $\bar{\lambda}_{LT} = \sqrt{\dfrac{W_y f_y}{M_{cr}}}$

For a class 1 section

$W_y = W_{pl,y}$

Under the given pattern of loading, *LTBeam* gives $M_{cr} = 320\text{ kNm}$

3-6/§6.3.2.2 Had there been no elastomeric pad between the rail and the beam, the vertical load could have been assumed to act at the level of the shear centre and M_{cr} would be 455 kNm.

3-1-1/§6.3.2.1(1) $\bar{\lambda}_{LT} = \sqrt{\dfrac{2610 \times 10^{-6} \times 265 \times 10^{3}}{320}} = \sqrt{\dfrac{692}{320}} = 1.47$

3-1-1/NA.2.17 For a UKB with $\dfrac{h}{b} > 2$ use buckling curve c

3-1-1/§6.3.2.3(1) Buckling curve c for $\bar{\lambda}_{LT} = 1.47$ gives:

$\chi_{LT} = 0.401$

The UK National Annex to BS EN 1993-6 refers to the NA to BS EN 1993-1-1 for partial factors γ_{M0} and γ_{M1} and this gives values of both equal to 1.0.

$M_{b,Rd} = \chi_{LT} M_{y,Rd}$

$M_{b,Rd} = 0.401 \times 692 = 277\text{ kNm}$

2.6.2 Combined bending and torsion resistance

Interaction between LTB, minor axis bending and torsion effects will be verified using BS EN 1993-6 (A.1), re-expressed as:

Section 3.2

$$\frac{M_{y,Ed}}{M_{b,Rd}} + C_{mz}\frac{M_{z,Ed}}{M_{z,Rd}} + k_w k_{zw} k_\alpha \frac{M_{w,Ed}}{M_{w,Rd}} \leq 1$$

where:

C_{mz} is the equivalent uniform moment factor for bending about the z-axis, from EN 1993-1-1 Table B.3.

In this example, since two-point loading is not considered in Table B.3, it seems reasonable to take $C_{mz} = 0.95$ as for a parabolic bending moment diagram under uniformly distributed load.

$$\frac{M_{y,Ed}}{M_{b,Rd}} = \frac{133.5}{277} = 0.482$$

$$M_{z,Rd} = \frac{W_{pl,z} f_y}{\gamma_{M0}}$$

$$= \frac{399\times10^{-6}\times265\times10^3}{1} = 106 \text{ kNm}$$

$$\frac{M_{z,Ed}}{M_{z,Rd}} = \frac{9.2}{106} = 0.09$$

$$M_{w,Rd} = \frac{W_{pl,f} f_y}{\gamma_{M0}} = \frac{\left(0.0174\times0.21^2/4\right)\times265\times10^3}{1} = 50.8 \text{ kNm}$$

$$\frac{M_{w,Ed}}{M_{w,Rd}} = \frac{2.33}{50.8} = 0.05$$

$$k_w = 0.7 - \frac{M_{w,Ed}}{M_{w,Rd}}$$

$$= 0.7 - 0.05 = 0.65$$

$$k_{zw} = 1 - \frac{M_{z,Ed}}{M_{z,Rd}}$$

$$= 1 - 0.09 = 0.91$$

$$k_\alpha = \frac{1}{\left(1-\frac{M_{y,Ed}}{M_{cr}}\right)} = \frac{1}{\left(1-\frac{133.5}{320}\right)} = 1.72$$

The interaction expression is evaluated as:

$0.48 + 0.95 \times 0.09 + 0.65 \times 0.91 \times 1.72 \times 0.05 = 0.48 + 0.09 + 0.05 = 0.62 < 1$

2.7 Serviceability limit state

2.7.1 Displacements

Maximum rotation in service will be between the wheels, not under them. Also, it may well be experienced when the crane is in a different position from the one which generates maximum moment. The same is true of the rotation under a wheel, if that is taken as the design criterion.

3-6/§7.3
3-6/Table 7.1

According to BS EN 1993-6, limits for deformations and displacements should be agreed for each project. The limiting value of horizontal deformation of $L/600$ given in BS EN 1993-6 for the SLS characteristic combination of actions is considered here.

For the ULS combination of actions, $\phi = 0.0146$ rad, under the left-hand wheel.

1-3/Table A.1, NA.2.6

The partial factors on permanent actions and crane actions at ULS are both equal to 1.35 according to BS EN 1991-3 and the UK NA. Hence the SLS deflection is 1/1.35 times the ULS deflection.

Assuming that the central deflection is 5% greater than at the left-hand wheel, the SLS rotation at mid-span is:

$0.0146 \times 1.05/1.35 = 0.0114 \text{ rad } (0.65°)$

At top of rail level, the displacement due to twist is $0.0114 \times 0.333 = 0.0038$ m (3.8 mm).

To determine the horizontal displacement of the beam under the influence of the two horizontal forces, for simplicity consider the central deflection due to two symmetrically placed loads at a distance d from the ends:

$$w = FL^3\,[3d/4L - (d/L)^3]/6EI$$

Here $d = (7.5 - 3)/2 = 2.25$ m

The force should be taken as that from the crane plus a component of the vertical load due to the rotation:

$$F = 3/1.35 + 50/1.35 \times 0.0114 = 2.64 \text{ kN}$$

$$w = \frac{2640 \times 7500^3}{6 \times 210000 \times 2690 \times 10^4}\left[\frac{3 \times 2250}{7500} - \left(\frac{2250}{7500}\right)^3\right] = 6.5 \text{ mm}$$

Total lateral displacement at rail level at mid-span:

$$w = 3.8 + 6.5 = 10.3 \text{ mm}$$

Limit $= L/600 = 7500/600 = 12.5$ mm OK

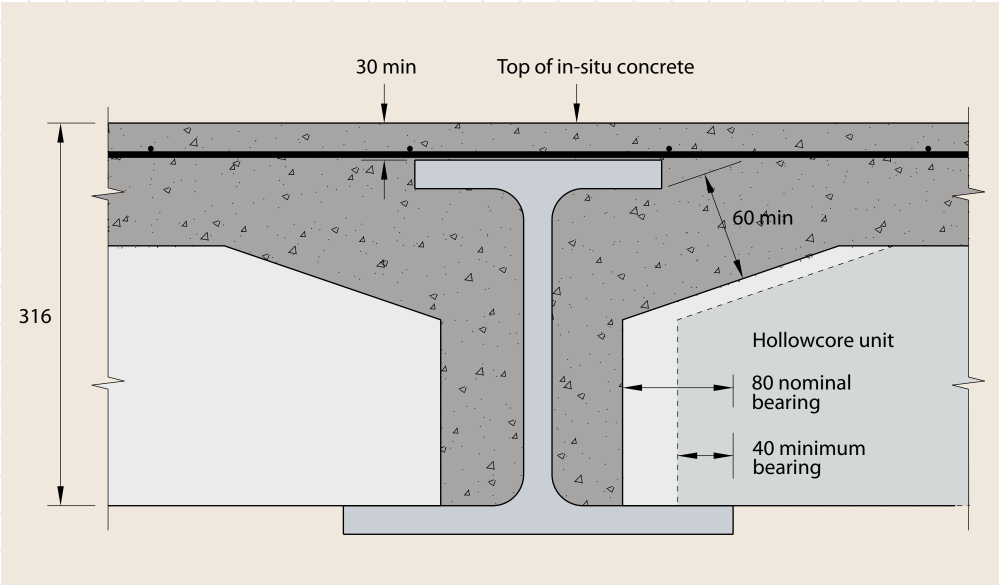
Example 3 - ASB at various stages of construction

3.1 Configuration

A floor is constructed using 300 ASB153 beams spanning 7.5 m and spaced at 7.5 m centres. The ASBs support 250 mm thick hollowcore units with in-situ lightweight concrete topping to a total concrete thickness of 316 mm.

Verify the beam at various stages of execution, with precast units on one and both sides and with construction loads on one and both sides.

The weight of the hollowcore units is 3.4 kN/m^2 (taken from manufacturer's literature) and the density of the wet lightweight concrete topping is taken as 19 kN/m^3 (corresponding to a dry density of 1750 kg/m^3).



3.2 Section properties

For 300 ASB 153 in S355:

P363 ASB is a Class 1 section

h = 310 mm

b_t = 190 mm

b_b = 300 mm

t_f = 24 mm

t_w = 27 mm

A = 196 cm^2

$W_{pl,y}$ = 2160 cm^3

I_z = 6840×10^{-8} m^4

Appendix A I_T = 513×10^{-8} m^4

I_w = 0.895×10^{-6} m^6

$e_{s,bf}$ = 58 mm (shear centre to mid bottom flange)

$e_{s,c}$ = 66 mm (shear centre to centroid)

a = 0.672 m

BS EN 10025-2 For $t > 16$ mm and S355

$f_y = R_{eH} = 345\ \text{N/mm}^2$

3.3 Actions

3.3.1 *Permanent actions*

Self weight of precast units $g_{k,1} = 3.4\ \text{kN/m}^2$

Self weight of beam $g_{k,2} = 1.5\ \text{kN/m}$

3.3.2 *Variable actions*

The weight of wet concrete q_{cf} is treated as a variable action:

$q_{cf} = 0.066 \times 19 = 1.25\ \text{kN/m}^2$

1-1-6/NA.2.13 BS EN 1991-1-6 NA.2.13 provides recommended values for q_{cc} and q_{ca} but allows alternative values to be determined.

q_{cc} is the construction load due to non-permanent equipment in position for use during execution

q_{ca} is the construction load due to working personnel, staff and visitors, possibly with hand tools or other small site equipment

Advisory Note AD 346 For composite beam design, AD 346 recommends the use of:

$q_{cc} = 0$ ($Q_{k,1a}$ in the AD)

$q_{ca} = 0.75\ \text{kN/m}^2$ ($Q_{k,1b}$ in the AD)

Total variable action $q_k = 2\ \text{kN/m}^2$

BS EN 1990

3.3.3 *Partial factors for actions*

Table NA.A1.2(B) Permanent actions $\gamma_G = 1.35$

Variable actions $\gamma_Q = 1.5$

3.3.4 *Combinations of actions for ULS*

BS EN 1990 Annex A.1 of BS EN 1993-1-6 recommends the use of $\psi_o = 1.0$ which results in expression 6.10a governing. Using the UK National Annex to BS EN 1990 results in expression 6.10a giving the same design values as expression 6.10 for the construction stages.

Table NA.A1.2(B) Here:

$f_d = \gamma_G g_{k,1} \times b + \gamma_G g_{k,2} + \gamma_Q q_k \times b$

where b is the spacing of the beams

Consider three design situations:

1. One side of beam loaded (precast units and construction loads)

   $f_d \quad = 1.35 \times 3.4 \times 3.75 + 1.35 \times 1.5 + 1.5 \times 2.0 \times 3.75 = 30.5 \text{ kN/m}$

   Note: this situation, where the concrete is placed on one side before the units are in place on the other, would normally be excluded by the method statement. However, it could arise if there were a construction opening on one side and since it demonstrates the worst torsional loading, is considered here.

2. One side of beam loaded with precast units and construction loads, the other with precast units only

   $f_d \quad = 1.35 \times 3.4 \times 7.5 + 1.35 \times 1.5 + 1.5 \times 2.0 \times 3.75 = 47.7 \text{ kN/m}$

3. Both sides of beam loaded with precast units and construction loads

   $f_d \quad = 1.35 \times 3.4 \times 7.5 + 1.35 \times 1.5 + 1.5 \times 2.0 \times 7.5 = 59.0 \text{ kN/m}$

3.4 Design values of vertical bending moments & shear forces at ULS

The major axis bending moment at mid-span and the shear force at the ends are given by:

$$M_{y,Ed} = \frac{f_d L^2}{8} \text{ and } V_{Ed} = \frac{f_d L}{2}$$

The values for the three situations are thus:

1. $M_{y,Ed} = 214 \text{ kNm}$ and $V_{Ed} = 114 \text{ kN}$
2. $M_{y,Ed} = 335 \text{ kNm}$ and $V_{Ed} = 179 \text{ kN}$
3. $M_{y,Ed} = 415 \text{ kNm}$ and $V_{Ed} = 221 \text{ kN}$

3.5 Design values of torsional effects at ULS

3.5.1 Total torque

Conservatively, assume that the bearing zone is reduced to 40 mm (to allow for tolerance in unit length and placement) on ‘Side 1’ and is the nominal 80 mm on ‘Side 2’. Also, assume that all the wet concrete and construction load is transferred through the bearing of the precast unit. The eccentricity of the reaction is thus 130 mm and 110 mm on the two sides, respectively. (This is a very conservative assumption, for illustrative purposes in this guide; with good site control the eccentricities would usually be assumed to be equal on both sides.)

The torque due to the precast units is:

$$T_{p1} = 1.35 \times 3.4 \times 3.75 \times 0.130 = 16.8 \text{ kNm}$$

T_{p2} $= -1.35 \times 3.4 \times 3.75 \times 0.110 = -14.2$ kNm

The torque due to the construction loads is:

T_{c1} $= 1.5 \times 2.0 \times 3.75 \times 0.130 = 11.0$ kNm

T_{c2} $= -1.5 \times 2.0 \times 3.75 \times 0.110 = -9.3$ kNm

The net torque for each design situation is thus:

1. T_d $= 16.8 + 11.0 = 27.8$ kNm
2. T_d $= 16.8 + 11.0 - 14.2 = 13.6$ kNm
3. T_d $= 16.8 + 11.0 - 14.2 - 9.3 = 4.3$ kNm

3.5.2 Simplified assessment of torsional effects

Since the ASB is relatively stiff in St Venant torsion, the simplification of ignoring it and determining warping moments directly is not pursued in this example.

3.5.3 Assessment of effects, allowing for interaction between St Venant torsion and warping torsion

The flanges are unrestrained against warping at their ends and the beam is subject to a uniformly distributed torque; graphs C and D in Appendix D and the expressions in Case 4 in Appendix C are thus applicable.

$$\frac{L}{a} = \frac{7.5}{0.672} = 11.1$$

This is beyond the range of the graphs, so use the expressions.

For Case 4, with $x = L/2$ (i.e. mid-span):

$$\phi = \frac{T_d a}{GI_T} \times \frac{a}{L}\left[\frac{\left(xL - x^2\right)}{2a^2} + \cosh\frac{x}{a} - \tanh\frac{L}{2a}\sinh\frac{x}{a} - 1\right]$$

For design situation (1) – maximum torque:

$$\phi = \frac{T_d \times 0.674}{GI_t} \times \frac{1}{11.1}\left[\frac{11.1^2}{8} + \cosh\frac{11.1}{2} - \tanh\frac{11.1}{2}\sinh\frac{11.1}{2} - 1\right]$$

$$= \frac{T_d a}{GI_t} \times 1.30 = \frac{27.8 \times 0.674 \times 1.3}{81 \times 10^6 \times 513 \times 10^{-8}} = 0.059 \text{ rad } (3.4°)$$

A strong case can be made for discounting rotation induced weak direction bending in this example, since the load is not freely suspended. The precast unit can provide the small horizontal force necessary to keep its reaction parallel to the rotated beam axis.

For Case 4, with $x = L/2$ (i.e. mid-span):

$$\phi'' = \frac{T_d}{GI_T a} \times \frac{a}{L}\left[-1 + \cosh\frac{x}{a} - \tanh\frac{L}{2a}\sinh\frac{x}{a}\right]$$

$$\phi'' = \frac{T_d}{GI_T a} \times 11.1\left[-1 + \cosh\frac{11.13}{2} - \tanh\frac{11.1}{2}\sinh\frac{11.1}{2}\right]$$

$$= -\frac{T_d}{GI_T a} \times 0.089$$

The warping moment in the top flange is given by:

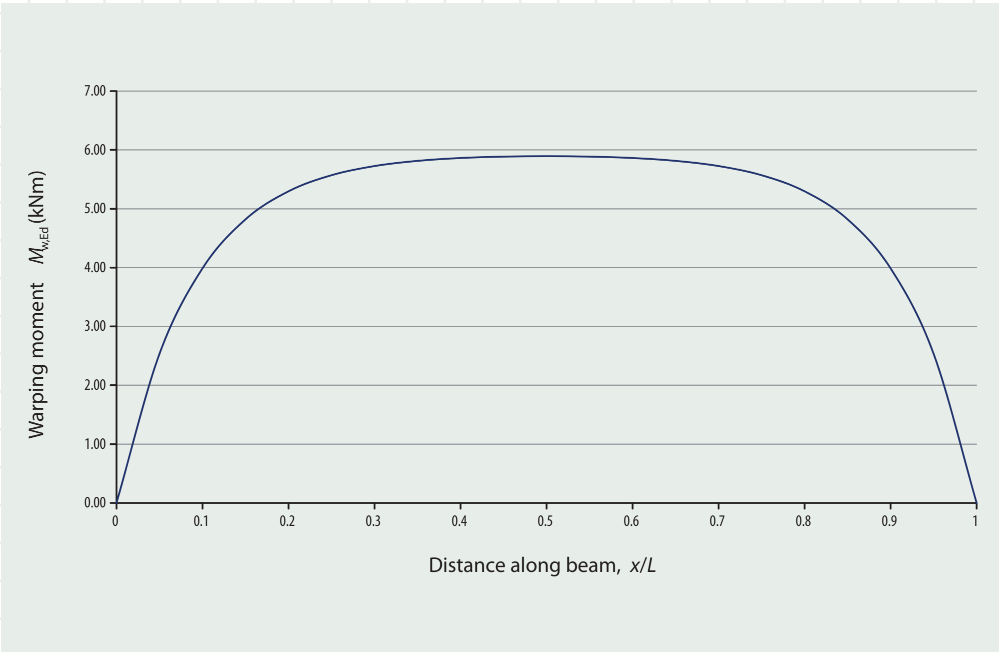
$$M_{w,Ed} = EI_w\phi''/(h - t_f) = \frac{EI_w T_d}{GI_T a(h - t_f)} \times 0.089 = \frac{T_d a}{(h - t_f)} \times 0.089$$

$$M_{w,Ed} = \frac{27.8 \times 0.674 \times 0.089}{(0.310 - 0.024)} = 5.89 \text{ kNm}$$ (for design situation (1))

Note: the warping moment in the bottom flange will be equal and opposite, but obviously the top flange will govern.

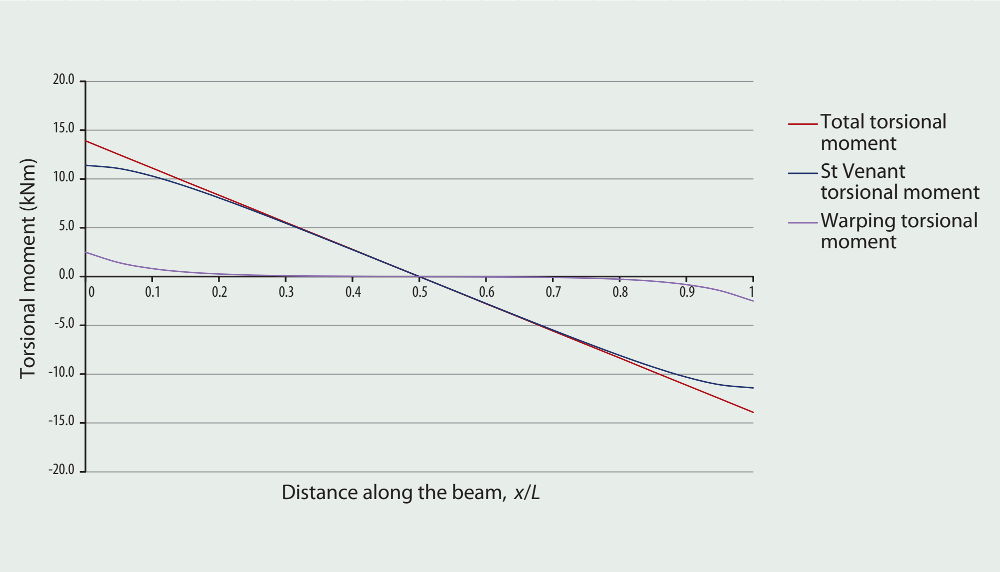
Commentary: Evaluating the warping moment (for design situation (1)) along the beam would give the following bending moment diagram:

Warping moment in flanges



It is not normally considered necessary to evaluate ϕ' and the St Venant torsional moment. However, for illustrative purposes, the variation of torsional moment along the beam is shown below.

The diagram shows that, as anticipated in neglecting the simplified assessment by warping alone, the majority of the torsion is resisted as St Venant torsional moments.



Variation of torsional moment along the beam

The effects for the three design situations are summarized below:

SITUATION	$M_{y,Ed}$ (kNm)	T_d (kNm)	$M_{w,Ed}$ (kNm) (mid-span)	$T_{t,Ed}$ (kNm) (at support)
1. Precast units and construction loads on one side of beam	214	27.8	5.89	11.4
2. Precast units both sides, construction loads on one side of beam	335	13.6	2.88	5.58
3. Precast units and construction loads on both sides of beam	415	4.3	0.91	1.76

3.6 Cross sectional resistance

3.6.1 Bending resistance

For this Class 1 section, the bending resistance about the major axis is:

3-1-1/§6.2.5(2)

$$M_{y,Rd} = \frac{W_{pl,y}\, f_y}{\gamma_{M0}}$$

γ_{M0} = 1.0, according to the UK NA

$$M_{y,Rd} = \frac{2160 \times 10^{-6} \times 345 \times 10^{3}}{1.0} = 745 \text{ kNm}$$

The maximum vertical bending moment on the steel section occurs in situation (3):

$$M_{y,Ed} = 415 \text{ kNm} < M_{y,Rd} = 745 \text{ kNm}$$

The warping resistance of the top flange is its plastic bending resistance:

Table A.4 $M_{w,Rd} \quad = \dfrac{W_{pl,z,tf} f_y}{\gamma_{M0}} = \dfrac{217 \times 345 \times 10^3}{1.0} = 74.8 \text{ kNm}$

The maximum warping moment on the steel section occurs in design situation (1):

$M_{w,Ed} \quad = 5.89 \text{ kNm} < M_{w,Rd} = 74.8 \text{ kNm}$

Consider the plastic interaction criterion, as given in Section 3.1.2:

$$\left[\frac{M_{y,Ed}}{M_{pl,y,Rd}}\right]^2 + \frac{M_{w,Ed}}{M_{pl,f,Rd}} + \frac{M_{z,Ed}}{M_{pl,z,Rd}} \leq 1$$

(This applies to the top flange)

Bending about the minor axis is not considered (as discussed above) hence the evaluation of this criterion is:

$$\left[\frac{415}{745}\right]^2 + \frac{5.89}{74} + 0 = 0.31 + 0.08 + 0 = 0.39 \leq 1 \text{ OK}$$

3.6.2 *Shear resistance*

Plastic shear resistance

Without torsion, the plastic shear resistance of the beam is given by:

$V_{pl,Rd} \quad = \dfrac{A_v f_y / \sqrt{3}}{\gamma_{M0}}$

For an ASB section:

$A_v \quad = A - b_t t_f - b_b t_f + (t_w + 2r) t_f$

$\quad = 19500 - 190 \times 24 - 300 \times 24 + (24 + 2 \times 27) \times 24 = 9610 \text{ mm}^2$

$V_{pl,Rd} \quad = \dfrac{9610 \times 345 / \sqrt{3}}{1.0} \times 10^{-3} = 3320 \text{ kN}$

This resistance is significantly greater than V_{Ed} for all three design situations.

Reduced shear resistance in the presence of torsion

The shear resistance is reduced by the presence of St Venant torsional shear stress in the web.

In this case, the reductions due to the St Venant stresses will not be critical but the maximum value will be calculated, for illustrative purposes.

$\tau_{t,Ed} \quad = T_{t,Ed} \times t / I_T$

$= 11.4 \times 106 \times 24/513 \times 10^4 = 53\ \text{N/mm}^2$ (for Case 1)

The reduction factor $= \sqrt{1 - \dfrac{\tau_{t,Ed}}{1.25 f_y / \sqrt{3}}} = \sqrt{1 - \dfrac{53\sqrt{3}}{1.25 \times 345}} = 0.89$

$V_{pl,T,Rd} \quad = 0.89 \times 3320 = 2940\ \text{kN}$

Note that St Venant shear stress does not, according to BS EN 1993-1-1, reduce the plastic bending resistance of an I section.

3.7 Buckling resistance

3.7.1 Lateral torsional buckling

It is assumed that, during construction, the wet concrete provides no restraint to the ASB and thus it can buckle in a LTB mode, in which the top flange displaces laterally. It is also assumed that friction at the underside of the precast units provides sufficient lateral restraint to prevent bending about the minor axis but does not influence the buckling resistance.

The buckling resistance is given by:

3-1-1/§6.3.2.1(1)

$$M_{b,Rd} = \chi_{LT} \frac{W_y f_y}{\gamma_{M1}}$$

where:

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \lambda_{LT}^2}} \quad \text{but } \chi_{LT} \leq 1.0$$

$$\Phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0.2\right) + \bar{\lambda}_{LT}^2\right]$$

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}}$$

To calculate M_{cr}, the elastic critical moment for lateral torsional buckling, *LTBeam* software can be used. At the time of writing, ASB sections have yet to be added to its catalogue but dimensions of the ASB sections may be directly entered.

In *LTBeam*, the loads can be applied at different levels. In this case, the reactions at the ends of the precast units (which include the reactions due to construction loads) are applied at the top of the bottom flange (which is $58 - 24/2 = 46$ mm below the shear centre) and the self weight of the beam is applied at the beam centroid (which is 66 mm above the shear centre).

The lowest value of M_{cr} is for situation (1) (slightly higher values would be given for situations (2) and (3) but for convenience, the lowest value will be used in all situations). From *LTBeam*:

M_{cr} = 1135 kNm

From this, the non dimensional slenderness is calculated as:

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}}$$

For this Class 1 section $W_y = W_{pl,y}$

$$\bar{\lambda}_{LT} = \sqrt{\frac{2160 \times 10^{-6} \times 345 \times 10^3}{1135}} = \sqrt{\frac{745}{1135}} = 0.810$$

For ASB sections, use buckling curve a (see Section 5.5)

3-1-1/Table 6.3 For buckling curve a, $\alpha_{LT} = 0.21$

3-1-1/§6.3.2.2(1) $\Phi_{LT} = 0.5\,[1 + 0.21(0.810 - 0.2) + 0.810^2] = 0.892$

$$\chi_{LT} = \frac{1}{\left[0.892 + \sqrt{0.892^2 - 0.810^2}\right]} = 0.79$$

$$M_{b,Rd} = \frac{\chi_{LT} W_y f_y}{\gamma_{M1}} = \frac{0.79 \times 744}{1.0} = 587 \text{ kNm}$$

The maximum design bending moment is $M_{y,Ed} = 415\text{kNm}$ (situation 3), so the buckling resistance is satisfactory.

3.7.2 Interaction of LTB with minor axis bending and torsion

As discussed in Section 3.2, use the formula in Annex A of BS EN 1993-6. For $\gamma_{M0} = \gamma_{M1}$ this may be re-expressed as:

$$\frac{M_{y,Ed}}{M_{b,Rd}} + \frac{C_{mz} M_{z,Ed}}{M_{z,Rd}} + \frac{k_w k_{zw} k_\alpha M_{w,Ed}}{M_{w,Rd}} \leq 1$$

Here $M_{z,Ed} = 0$, as discussed above, and thus $k_{zw} = 1$

$$k_w = 0.7 - 0.2\frac{M_{w,Ed}}{M_{w,Rd}}$$

$$k_\alpha = \frac{1}{1 - \left(\frac{M_{y,Ed}}{M_{cr}}\right)}$$

For situation (1), $M_{y,Ed} = 214$ kNm, $M_{w,Ed} = 5.89$ kNm

$$\frac{M_{y,Ed}}{M_{b,Rd}} = \frac{214}{587} = 0.36$$

$$\frac{M_{w,Ed}}{M_{w,Rd}} = \frac{5.89}{74} = 0.08$$

$$k_w = 0.7 - 0.2 \times 0.08 = 0.68$$

$$k_\alpha = \frac{1}{1-(214/1128)} = 1.23$$

The criterion is evaluated as:

$0.36 + 0.68 \times 1.23 \times 0.08 = 0.36 + 0.07 = 0.43 < 1$ OK

For situation (3), $M_{y,Ed} = 415$ kNm, $M_{w,Ed} = 0.91$ kNm

$$\frac{M_{y,Ed}}{M_{b,Rd}} = \frac{415}{587} = 0.71$$

$$\frac{M_{w,Ed}}{M_{w,Rd}} = \frac{0.91}{74} = 0.01$$

$$k_w = 0.7 - 0.2 \times 0.01 = 0.70$$

$$k_\alpha = \frac{1}{1-(415/1128)} = 1.58$$

The criterion is evaluated as:

$0.71 + 0.70 \times 1.58 \times 0.01 = 0.71 + 0.01 = 0.72 < 1$ OK

Situation (2) has intermediate values of $M_{y,Ed}$ and $M_{w,Ed}$ and is also satisfactory, by inspection.

3.8 Serviceability limit state

At SLS the partial factors on actions are both unity and thus the total torques are:

1. $16.8/1.35 + 11/1.5 = 12.4 + 7.3 = 19.7$ kNm
2. $19.7 - 14.2/1.35 = 9.2$ kNm
3. $9.2 - 9.3/1.5 = 2.9$ kNm

The rotations are thus:

1. $19.7/27.8 \times 0.059 = 0.042$ rad (2.4°)
2. $9.2/27.8 \times 0.059 = 0.020$ rad (1.1°)
3. $2.9/27.8 \times 0.059 = 0.006$ rad (0.4°)

If no slab were to be placed on the second side (i.e. the ASB were an edge beam or adjacent to an opening) most of the situation (1) rotation would be locked in when the concrete hardened.

In most situations, the precast units would be placed on the second side before any concrete is cast and thus only the situation (2) rotation would be locked in, should one side be cast before the other, or only the situation 3 rotation (if both were cast at the same time).

At the maximum rotation, in situation (1), the mid-span deflection of the top flange, assuming that bottom flange is restrained by the precast units, would be:

$0.042 \times (310 - 24) = 12$ mm

The recommended limit to lateral deflection (see Section 5.6) is:

$L/500 \quad = 7500/500 = 15$ mm

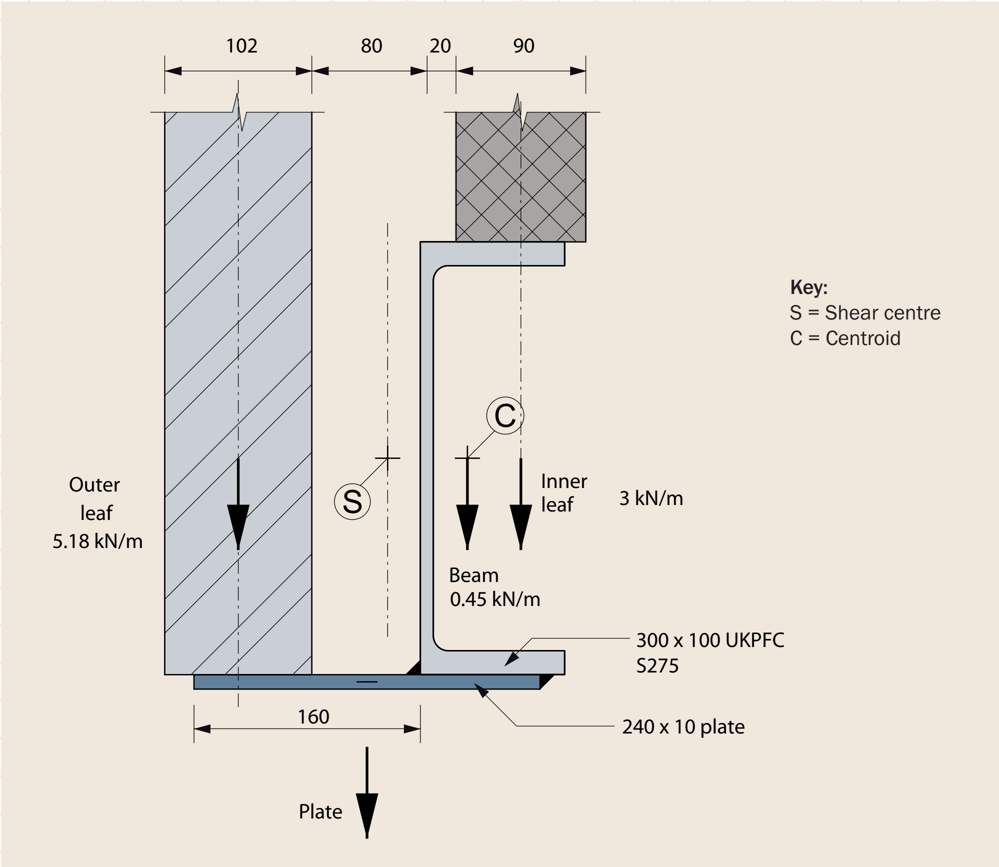
The maximum deflection is within that limit.

Example 4 - Lintel in Cavity Wall using a UKPFC

4.1 Configuration

A 300 × 100 UKPFC acts as a lintel to support a 2 m high cavity wall. A 240 × 10 plate welded to the underside of the channel supports the outer leaf, but will not be considered to act compositely.

The beam is laterally unrestrained over its effective span of 5 m. Assume that each end is restrained against torsion but not against warping.



4.2 Section properties

SCI P363 For a 300 × 100 × 46 UK PFC

t_f = 16.5 mm

t_w = 9 mm

h = 300 mm

b = 100 mm

r = 15 mm

I_T = 36.8 × 10^4 mm^4

i_z = 31.3 mm

$W_{pl,y}$ = 641 cm^3

A = 58.0 × 10^2 mm^2

Appendix A	e_0	$= 36.8$ mm (from centreline of web)
	$e_{s,c}$	$= 62.7$ mm (shear centre to centroid)
	I_w	$= 81.3 \times 10^9$ mm^6
	a	$= 765$ mm

For $t_f > 16$ mm and S275

BS EN 10025-2 $f_y = R_{eH} = 265$ N/mm^2

4.3 Actions

The permanent loads on the beam are:

Outer leaf:	2.3 m height of brickwork at 2.25 kN/m^2	$g_{k,1}$	=	5.18 kN/m
Inner leaf:	2.0 m height of blockwork at 1.5 kN/m^2	$g_{k,2}$	=	3 kN/m
Beam:	46 kg/m	$g_{k,3}$	=	0.45 kN/m
Plate:	$0.24 \times 0.01 \times 7850 = 18.8$ kg/m	$g_{k,4}$	=	0.18 kN/m
	Total:	$g_{k,tot}$	=	8.81 kN/m

Taking moments about the shear centre (clockwise positive), the torques are:

Outer leaf:	$5.18 \times (-0.131 - 0.0045 + 0.0367)$	$= -0.512$ kNm/m
Inner leaf:	$3 \times (0.065 - 0.0045 + 0.0367)$	$= 0.292$ kNm/m
Beam:	0.45×0.0627	$= 0.028$ kNm/m
Plate:	$0.18 \times (-0.040 - 0.0045 + 0.0367)$	$= -0.001$ kNm/m
	Total torque:	$= -0.193$ kNm/m

Partial factor for permanent actions

BS EN 1990 Table NA.A1.2 $\gamma_G = 1.35$

4.4 Combination of actions

BS EN 1990 As there are only permanent actions present, equation 6.10(a) will be more onerous than 6.10(b) and will therefore govern. The design value for the combination of actions at ULS is:

$$f_d = \Sigma\gamma_G g_{k,i} = 1.35 g_{k,tot} = 1.35 \times 8.81 = 11.9 \text{ kN/m}$$

The design value of the total applied torque on the 5 m long beam is:

$$T_d = 1.35 \times 0.193 \times 5 = 1.30 \text{ kNm (acting anticlockwise)}$$

4.5 Design value of vertical bending moment and shear

Bending moment at mid-span (ULS):

$$M_{y,Ed} = \frac{11.9 \times 5^2}{8} = 37.2 \text{ kNm}$$

The design shear force at each support is:

$$V_{Ed} = \frac{11.9 \times 5}{2} = 30 \text{ kN}$$

4.6 Design value of torsional effects at ULS

4.6.1 *Simplified assessment of effects*

Consider the torsion resisted only by warping. The total torque is equivalent to a couple $F = 1.30/(h - t_f) = 1.30/0.2835 = 4.59$ kN.

This force acts as a UDL along each flange and thus the bending moment in each flange is given by:

$$M_{w,Ed} = \frac{4.59 \times 5}{8} = 2.87 \text{ kNm}$$

4.6.2 *Assessment of effects, allowing for elastic interaction between St Venant torsion and warping torsion*

The flanges are assumed to be unrestrained against warping at the member ends and thus Graphs C and D in Appendix D and the expressions in Case 4 in Appendix C are applicable.

For this beam:

$$\frac{L}{a} = \frac{5.0}{0.765} = 6.54$$

From Graph C, curve A, with $L/a = 6.54$:

$$\frac{\phi\, GI_T}{T_{Ed}\, a} = 0.675$$

From Graph D, curve A, with $L/a = 6.54$:

$$-\frac{\phi'' aGI_T}{T_{Ed}} = 0.14$$

whence:

$$\phi = \frac{0.675 \times 1.30 \times 0.765}{(81 \times 10^6 \times 36.8 \times 10^{-8})} = 0.0225 \text{ rad } (1.29°)$$

Minor axis moment due to rotation:

$$M_{z,Ed} = \phi M_{y,Ed} = 0.0225 \times 37.2 = 0.837 \text{ kNm}$$

$$\phi'' = \frac{0.140 \times 1.30}{(G \times 36.8 \times 10^{-8} \times 0.765)} = \frac{646 \times 10^3}{G}$$

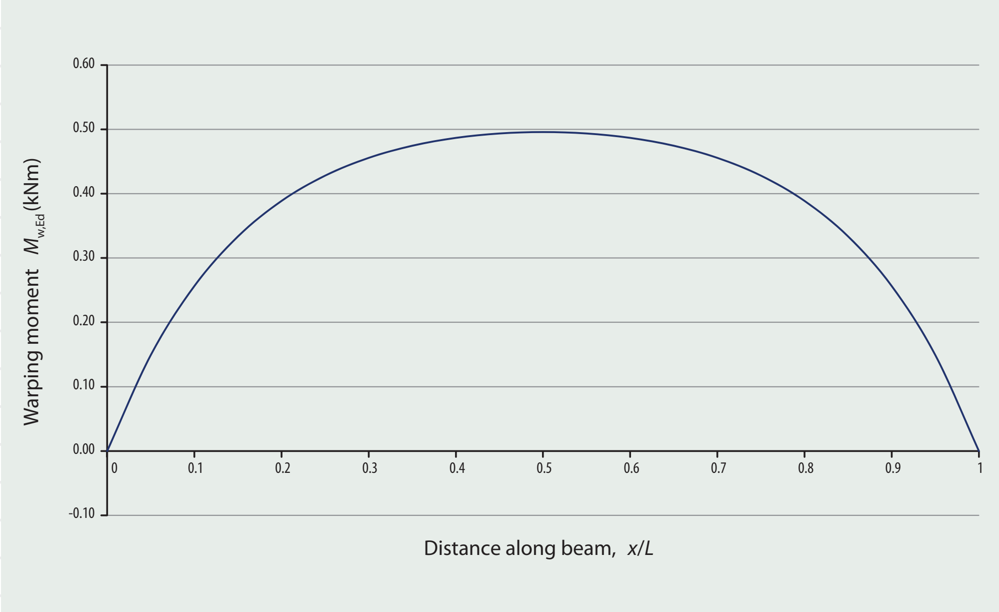
Warping moment $M_{\mathrm{w,Ed}} = \dfrac{\pm EI_{\mathrm{w}}\phi''}{(h - t_{\mathrm{f}})}$

Taking $E/G = 2.6$, this gives:

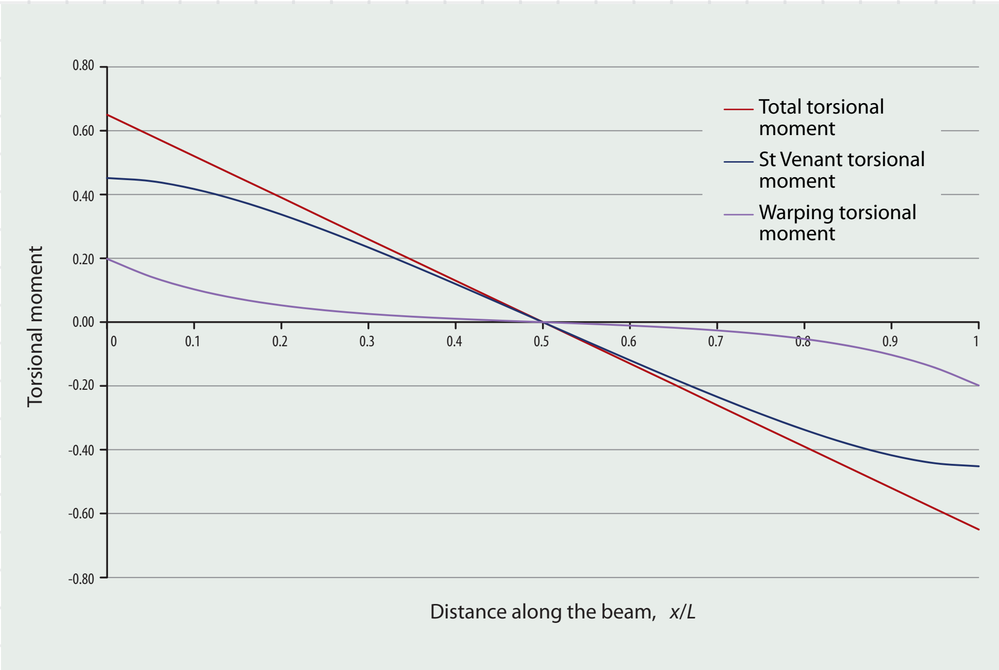
$$M_{\mathrm{w,Ed}} = \pm 2.6 \times \frac{8.13 \times 10^{-8} \times 646 \times 10^{3}}{(0.3 - 0.0165)} = 0.48 \text{ kNm}$$

The moment in the top flange ($= 0.48 + 0.84/2 = 0.90$ kNm) is significantly less than calculated by the simplified assessment (2.87 kNm).

Commentary: Evaluation of the expressions in Appendix D would give the variation of warping moment and torsional moments along the beam. They are shown below for information.



Warping moment in flanges of lintel



Variation of torsional moment along the beam

4.7 Cross sectional resistance

Partial factor for resistance

3-1-1/NA.2.15 $\gamma_{M0} \quad = 1.0$

4.7.1 Bending resistance

The channel is a Class 1 section.

For a Class 1 section, the bending resistance about the major axis is given by:

BS EN 1993-1-1 6.2.5(2)

$$M_{c,y,Rd} = \frac{W_{pl,y} f_y}{\gamma_{M0}} = \frac{641 \times 10^3 \times 265 \times 10^{-6}}{1.0} = 170 \text{ kNm}$$

$M_{c,y,Rd} > M_{y,Ed} = 37.2$ kNm

The bending resistance about the minor axis is:

$$M_{c,z,Rd} = \frac{W_{pl,z} f_y}{\gamma_{M0}} = \frac{148 \times 10^{-6} \times 265 \times 10^3}{1.0} = 39 \text{ kNm}$$

The warping resistance of one flange (on its own, without part of the web) is:

$$M_{w,Rd} = M_{w,Rd} = \frac{W_{f,pl} f_y}{\gamma_{M0}} = \frac{\left(100^2 \times 16.5/4\right) \times 265}{1.0} \times 10^{-6} = 10.9 \text{ kNm}$$

$M_{w,Rd} > M_{w,Ed} = 0.90$ kNm

4.7.2 Shear resistance

Shear would not be expected to be significant. However, with a channel section the need arises to consider web shear arising from warping as well as St Venant, so the opportunity is taken to demonstrate the procedure.

The elastic shear stress in the web due to warping is given by $\tau_t = -\dfrac{ES_{w2}\phi'''}{t_w}$

The shear stress in the web due to St Venant is given by $\tau_t = Gt_w \phi'$.

For this simply supported beam, both effects are greatest at the supports (where $x = 0$). The expressions from Appendix C, when $x = 0$ are:

$$\phi' = \frac{T_{Ed}}{GI_T}\left(\frac{a}{L}\right)\left\{\frac{L}{2a} - \tanh\left(\frac{L}{2a}\right)\right\}$$

$$\phi''' = \frac{T_{Ed}}{GI_T a^2}\left(\frac{a}{L}\right)\left\{-\tanh\left(\frac{L}{2a}\right)\right\}$$

Hence:

$$\phi' = \left(\frac{1.30}{81\times10^{6}\times36.8\times10^{-8}}\right)\times\left(\frac{0.765}{5}\right)\times\left\{\frac{5.0}{2\times0.765}-\tanh\left(\frac{5.0}{2\times0.765}\right)\right\}$$

$$= 0.015 \text{ rad/m}$$

Therefore, the shear stress due to St Venant torsion is:

$$\tau_{t,Ed} = G\,t_w\,\phi' = 81\times10^{3}\times9\times0.015\times10^{-3} = 10.9 \text{ N/mm}^2$$

And

$$\phi''' = \left(\frac{1.30}{81\times10^{6}\times36.8\times10^{-8}\times0.765}\right)\times\left(\frac{0.765}{5.0}\right)\times\left\{-\tanh\left(\frac{5.0}{2\times0.765}\right)\right\}$$

$$= -0.011 \text{ rad/m}^3$$

The shear stress due to warping torsion is:

$$\tau_{t,Ed} = \frac{ES_{w2}\phi'''}{t_w}$$

Appendix A where S_{w2} = 260 cm^4 ($S_{w2} > S_{w3}$)

$$\tau_{w,Ed} = \frac{-210\times10^{3}\times260\times10^{4}\times(-0.011\times10^{-9})}{9} = 0.67 \text{ N/mm}^2$$

The shear stresses will reduce the vertical shear resistance of the web.

(6.27) The reduction factor applied to $V_{pl,Rd}$ is:

$$\left[\sqrt{1-\frac{\tau_{t,Ed}}{1.25 f_y/\sqrt{3}}}-\frac{\tau_{w,Ed}}{f_y/\sqrt{3}}\right]$$

$$= \sqrt{1-\frac{10.9\sqrt{3}}{1.25\times265}}-\frac{1\sqrt{3}}{265}$$

$$= 0.97 - 0.01 = 0.96$$

The shear resistance in the absence of torsion is:

$$V_{pl,Rd} = \frac{A_v f_y/\sqrt{3}}{\gamma_{M0}}$$

Where, for a channel section:

$A_v = A - 2bt_f + b_f(t_w + r)$

$= 58 \times 10^2 - (2 \times 100 \times 16.5) + 16.5 \times (9 + 15) = 2896 \text{ mm}^2$

Therefore:

$$V_{pl,Rd} = \frac{2896 \times 265/\sqrt{3}}{1.0} \times 10^{-3} = 443 \text{ kN}$$

Thus, the shear resistance in the presence of torsion is:

$V_{pl,T,Rd} = 0.96 \times 443 = 425 \text{ kN} > V_{Ed} = 30 \text{ kN}$ Satisfactory

4.8 Buckling resistance

4.8.1 Lateral torsional buckling

Bending resistance of this unrestrained beam will be limited by lateral torsional buckling. The non-dimensional slenderness needs to be evaluated. For this example the load is not destabilizing and the conservative approximation in publication P362 will be used. (The *LTBeam* software used for the other examples in this publication is restricted to sections symmetrical about their z-axis.)

P362, Section 6.3.2.3

$$\bar{\lambda} \approx \bar{\lambda}_{LT} = \frac{L/i_z}{96} \text{ for S275}$$

where:

L is the effective span of the beam = 5 m

$$\bar{\lambda} = \frac{5}{0.0313 \times 96} = 1.664$$

3-1-1/NA.2.17 According to the NA to BS EN 1993-1-1, for hot rolled sections that are not doubly symmetric, buckling curve d should be used.

For $\bar{\lambda}_{LT} = 1.66$, $\chi_{LT} = 0.29$

3-1-1/6.3.2.3 In this case, with a large slenderness value, the modification factor in 3-1-1/§6.3.2.3(2) does not offer any enhancement.

Therefore,

$$M_{b,Rd} = \frac{\chi_{LT} W_y f_y}{\gamma_{M1}}$$

Since $\gamma_{M0} = \gamma_{M1}$ this can be expressed as $M_{b,Rd} = \chi_{LT} M_{c,Rd}$

Appendix E, 4.7.1 $M_{b,Rd} = 0.29 \times 170 = 49 \text{ kNm}$

4.8.2 Interaction of LTB with minor axis bending and torsion

As discussed in Section 3.2, use the formula in Annex A of BS EN 1993 6. For $\gamma_{M0} = \gamma_{M1}$ this may be re-expressed as:

$$\frac{M_{y,Ed}}{M_{b,Rd}} + C_m \frac{M_{z,Ed}}{M_{z,Rd}} + k_w\, k_{zw}\, k_d \frac{M_{w,Ed}}{M_{w,Rd}}$$

where:

BS EN 1993-1-1 Table B.3

C_m = 0.95 (for a simply supported beam with a uniform load)

Section 3.2

$$k_w = 0.7 - 0.2 \frac{M_{w,Ed}}{M_{w,Rd}}$$

$$k_{z,w} = 1 - \frac{M_{z,Ed}}{M_{z,Rd}}$$

$$k_\alpha = \frac{1}{1-\left(M_{y,Ed} / M_{cr}\right)}$$

In this case:

$$\frac{M_{w,Ed}}{M_{w,Rd}} = \frac{0.48}{10.9} = 0.04$$

$$\frac{M_{z,Ed}}{M_{z,Rd}} = \frac{0.84}{39} = 0.02$$

$$\frac{M_{y,Ed}}{M_{cr}} = \frac{37.2}{M_{cr}}$$

$$\frac{M_{y,Ed}}{M_{b,Rd}} = \frac{37.2}{49} = 0.76$$

Since M_{cr} has not been calculated whilst determining $\bar{\lambda}_{LT}$, it will be back calculated from:

BS EN 1993-1-1 6.3.2.2(1)

$$\bar{\lambda}_{LT} = \sqrt{\frac{M_y f_y}{M_{cr}}}$$

$$M_{cr} = \frac{W_y f_y}{\bar{\lambda}_{LT}^{\,2}}$$

For a Class 1 section, according to the UK NA to BS EN 1993-1-1:

$$W_y f_y = W_{pl,y} f_y = M_{y,Rd}$$

$$M_{cr} = \frac{170}{1.664^2} = 61.4 \text{ kNm}$$

Therefore:

$$k_w = 0.7 - (0.2 \times 0.03) = 0.69$$

k_{zw} $= 1 - 0.03 = 0.97$

k_α $= \dfrac{1}{1-(37.2/61.4)} = 2.54$

Therefore the criterion is evaluated as:

$$\frac{37.2}{49} + 0.95 \times 0.02 + 0.69 \times 0.97 \times 2.54 \times 0.04$$

$= 0.76 + 0.02 + 0.07 = 0.85$

$0.85 < 1.0$

Therefore, the resistance of the member to combined bending and torsion is satisfactory.

4.9 Serviceability limit state

The rotation at SLS will be $\phi = \dfrac{0.0225}{1.35} = 0.019$ rad, 1.06°.

If a complete prefabricated wall were placed on the beam, the top of the 2 m high wall would, at mid-span, lean 40 mm out from its intended position.

In reality, since the wall will be laid in courses, a considerable degree of compensation may be expected, especially if the bricklayer is aware of the potential problem. Also, if the two leaves are not brought up in parallel, a greater twist will occur when the outer leaf is constructed. The designer should consider whether any special requirements for verticality need to be specified.

The vertical deflection is:

$$w = \frac{5 f_{d,ser} L^4}{384 E I_y} = \frac{5 \times 8.81 \times 5000^4}{384 \times 210000 \times 8230 \times 10^4} = 4.1 \text{ mm}$$

There is no commonly accepted limit for deflection due to permanent actions but this modest deflection would normally be satisfactory.

4.10 Commentary

This traditional detail is probably better suited to shorter spans, because of the twist. Substitution of a RHS, or a proprietary closed section lintel would significantly reduce twist – see Example 5.

The simplified method of assessment, ignoring St Venant torsion, gives much greater warping moment than that determined by evaluating interaction (it is approximately six times greater) and the interaction would fail the limiting criterion. The rotation using that simplification has not been evaluated but would certainly be greater.

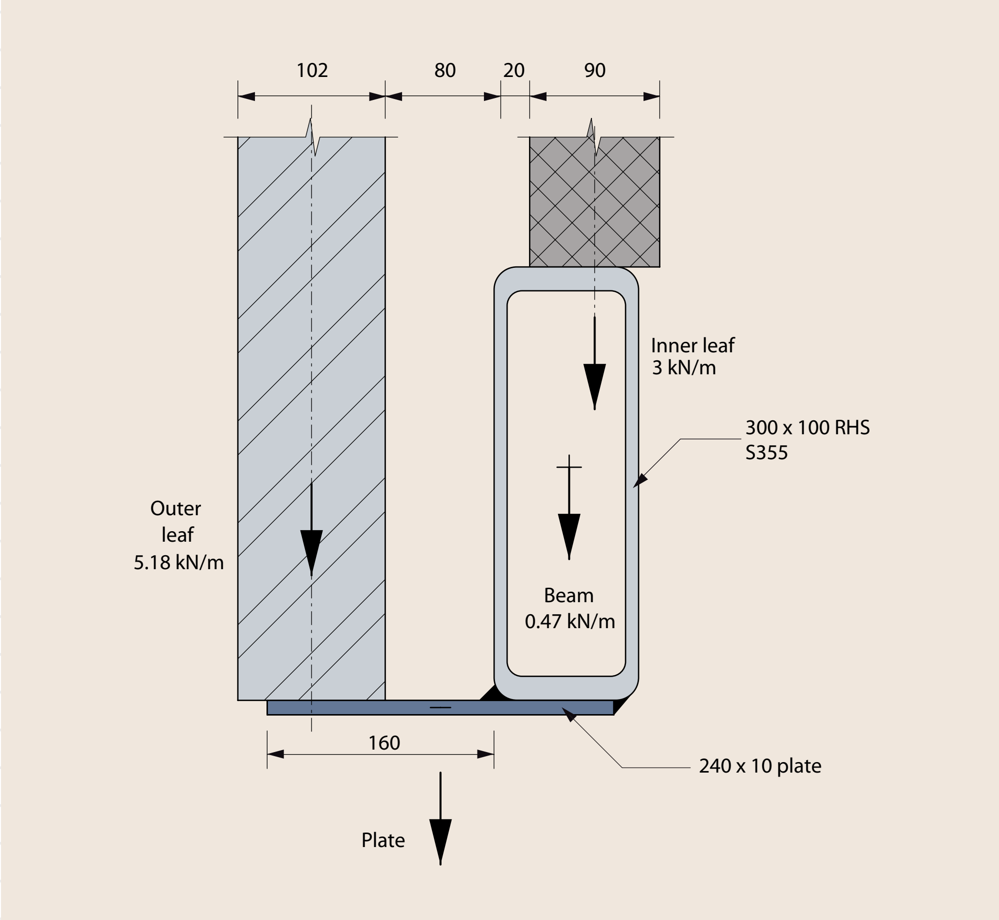
The stresses associated with the interacting effects are generally modest.

The plate welded to the bottom flange of the channel has been ignored in this assessment. If the ‘composite’ welded section were to be assessed, the calculation of torsional properties and effects would be quite complex; the position of the shear centre would move horizontally slightly (toward the channel) and downwards; since the warping resistance is controlled by the top flange, there would be very little effect on its value.

Example 5 - Lintel in Cavity Wall using a Hollow Section

5.1 Configuration

A 300 × 100 × 8 rectangular hollow section is used in place of the channel section in Example 4. The overall dimensions and self weight are very similar to those in Example 4.



5.2 Section properties

For a 300 × 100 × 8 RHS

h = 300 mm

b = 100 mm

t = 8 mm

I_T = 3070 cm^4

$W_{pl,y}$ = 546 cm^3

W_t = 387 cm^3

The shear centre and the centroid are both located at the middle of the RHS.

BS EN 10025-2 For $t_f < 16$ mm and S355

$f_y = R_{eH} = 355$ N/mm^2

5.3 Actions

The permanent loads on the beam, are:

Outer leaf:	2.3 m height of brickwork at 2.25 kN/m^2	$g_{k,1}$	=	5.18 kN/m
Inner leaf:	2.0 m height of blockwork at 1.5 kN/m^2	$g_{k,2}$	=	3 kN/m
Beam:	47.7 kg/m	$g_{k,3}$	=	0.47 kN/m
Plate:	$0.24 \times 0.01 \times 7850 = 18.8$ kg/m	$g_{k,4}$	=	0.18 kN/m
	Total:	$g_{k,tot}$	=	8.83 kN/m

Taking moments about the shear centre (clockwise positive), the torques are:

Outer leaf:	$5.18 \times (-0.181)$	$= -0.938$ kNm/m
Inner leaf:	3×0.015	$= 0.045$ kNm/m
Beam:	0.45×0	$= 0$
Plate:	$0.18 \times (-0.090)$	$= -0.016$ kNm/m
	Total torque:	$= -0.909$ kNm/m

Partial factor for permanent actions

BS EN 1990 Table NA.A1.2

$\gamma_G = 1.35$

5.4 Combination of actions

BS EN 1990

As there are only permanent actions present, equation 6.10(a) will be more onerous than 6.10(b) and will therefore govern. The design value for the combination of actions at ULS is:

$$f_d = \Sigma \gamma_G \, g_{k,i} = 1.35 \, g_{k,tot} = 1.35 \times 8.83 = 11.9 \text{ kN/m}$$

The design value of the total applied torque on the 5 m long beam is:

$$T_d = 1.35 \times 0.909 \times 5 = 6.14 \text{ kNm (acting anticlockwise)}$$

5.5 Design value of vertical bending moment

Bending moment at mid-span (ULS):

$$M_{y,Ed} = \frac{11.9 \times 5^2}{8} = 37.2 \text{ kNm}$$

5.6 Design values of torsional effects

5.6.1 Torsional moments

Consider the torsion resisted only by St Venant torsion (as allowed by BS EN 1993-1-1, 6.2.7(7)).

The design value of torsional moment is zero at mid-span, increasing linearly (but in opposite senses) to $T_{t,Ed} = T_{Ed} = 6.14/2 = 3.07$ kNm at each support.

5.6.2 Rotation

The average torque over each half of the beam is 3.07/2 = 1.54 kNm so rotation at mid-span is given by:

$\phi \quad = \dfrac{T_{\mathrm{av,t,Ed}}\,L/2}{GI_{\mathrm{T}}} = \dfrac{1.54\times 2.5}{81\times 10^{6}\times 3070\times 10^{-8}} = 0.00156 \text{ rad } (0.09°)$

The minor axis moment due to this very small rotation is negligible.

5.7 Cross sectional resistance

Partial factor for resistance:

$\gamma_{\mathrm{M0}} \quad = 1.0$

5.7.1 Bending resistance

The RHS is a Class 2 section in bending. For a Class 2 section:

3-1-1/§6.2.5(2) $M_{\mathrm{c,y,Rd}} \quad = \dfrac{W_{\mathrm{pl,y}}f_{\mathrm{y}}}{\gamma_{\mathrm{M0}}} = \dfrac{546\times 10^{-6}\times 355\times 10^{3}}{1.0} = 194 \text{ kNm}$

$M_{\mathrm{c,y,Rd}} \quad = 194 \text{ kN} > M_{\mathrm{y,Ed}} = 37.2 \text{ kN}$ OK

5.7.2 Torsion resistance

The design resistance to St Venant torsion is given by:

$T_{\mathrm{Rd}} \quad = \dfrac{W_{\mathrm{t}}f_{\mathrm{y}}/\sqrt{3}}{\gamma_{\mathrm{M0}}} = \dfrac{387\times 10^{-6}\times 355\times 10^{3}/\sqrt{3}}{1.0} = 79 \text{ kNm}$

The shear stress due to the St Venant torsional moment is:

$\tau \quad = T_{\mathrm{t,Ed}} / W_{\mathrm{t}} = 3.07 \times 10^{6} / 387 \times 10^{3} = 7.9 \text{ N/mm}^2$

5.7.3 Shear resistance

Shear resistance in the absence of torsion is:

3-1-1/§6.2.6(2) $V_{\mathrm{pl,Rd}} \quad = \dfrac{A_{\mathrm{v}}\,f_{\mathrm{y}}/\sqrt{3}}{\gamma_{\mathrm{M0}}}$

3-1-1/§6.2.6(3) For a RHS with the load applied parallel to the height,

$A_{\mathrm{v}} \quad = \dfrac{Ah}{b+h} = \dfrac{60.8\times 10^{-4}\times 0.3}{0.1+0.3} = 4.56 \times 10^{-3} \text{ m}^2$

$V_{\mathrm{pl,Rd}} \quad = \dfrac{4.56\times 10^{-3}\times 355\times 10^{3}/\sqrt{3}}{1.0} = 934 \text{ kN}$

The reduced shear resistance accounting for torsion is:

3-1-1/§6.2.7(9) $V_{pl,T,Rd} = \left[1 - \frac{\tau_{t,Ed}}{f_y/\sqrt{3}}\right] V_{pl,Rd} = \left[1 - \frac{7.9}{355/\sqrt{3}}\right] 934 = 898 \text{ kN}$

$V_{pl,T,Rd} = 898 > V_{Ed} = 30 \text{ kN}$ OK

5.7.4 Combined bending and torsion

The maximum values of design bending moment (M_{Ed}) and design torque (T_{Ed}) occur in different locations and thus no interaction between these values need be considered. At intermediate locations, there are combined effects of lesser design values; to verify the combination at all locations, one could conservatively consider the maximum values to be coexistent. However, the very low St Venant shear stress (7.9 N/mm^2 at maximum) would give rise to very little reduction in bending resistance (see Section 6.2) and the interaction at all locations is satisfactory by inspection.

5.8 Buckling resistance

The RHS section is not susceptible to lateral torsional buckling (see Section 6.2) and thus the buckling resistance (and interaction with torsion) does not need to be verified.

5.9 Serviceability limit state

The rotation at SLS will be $\phi = \frac{0.00156}{1.35} = 0.00116 \text{ rad } (0.066°)$, which is satisfactory.

$f_d = g_{k,tot} = 8.83 \text{ kN/m}$

The vertical deflection is:

$$w = \frac{5 f_d L^4}{384 E I_y} = \frac{5 \times 8.83 \times 5000^4}{384 \times 210000 \times 6310 \times 10^4} = 5.4 \text{ mm}$$

There is no common limit for deflection due to permanent actions but this modest deflection would normally be satisfactory.

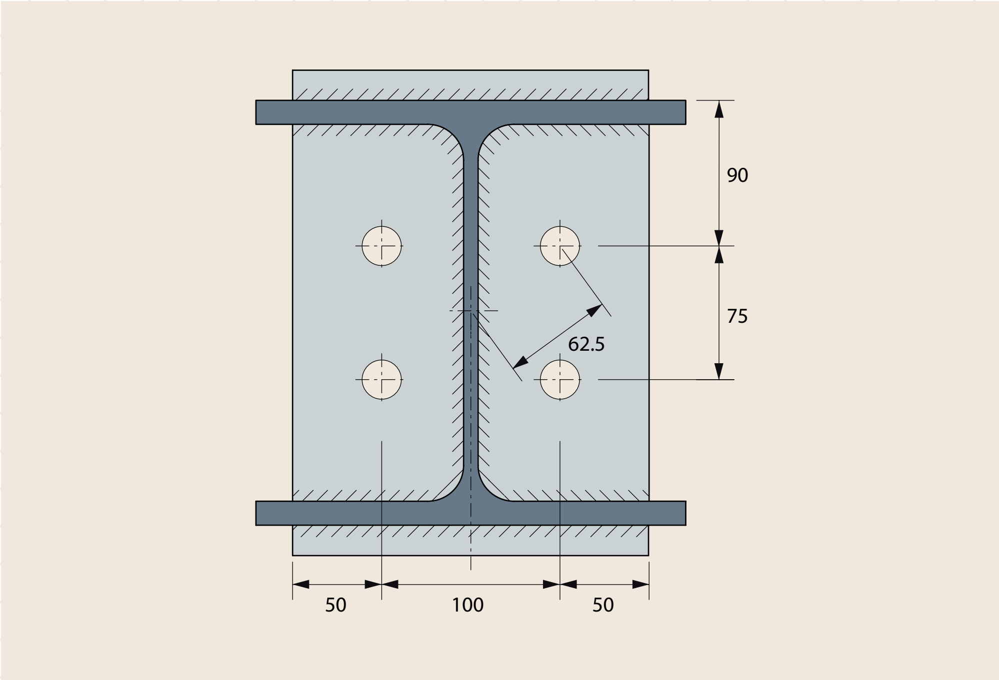
5.10 Commentary

The trial section is more than adequate; the rotation is very much less than that of the channel section in Example 4. A smaller, lighter RHS would suffice, although the vertical deflection would be greater and is likely to become the limiting criterion.

Example 6 - End Plate Connection

6.1 Configuration

Consider the simply supported beam of Example 1 with full depth end plate connections, as shown below:



The beam is a 254 UKC 73 with 10 mm end plate, grade S275.

Bolts are M20, class 8.8.

Two approaches are examined to verify the resistance of the end plate connection.

6.2 Design values of forces on connection

Vertical force $V_{\mathrm{Ed}} = 52 \text{ kN}$

Torque $T_{\mathrm{Ed}} = 3.75 \text{ kNm}$

6.3 Approach one

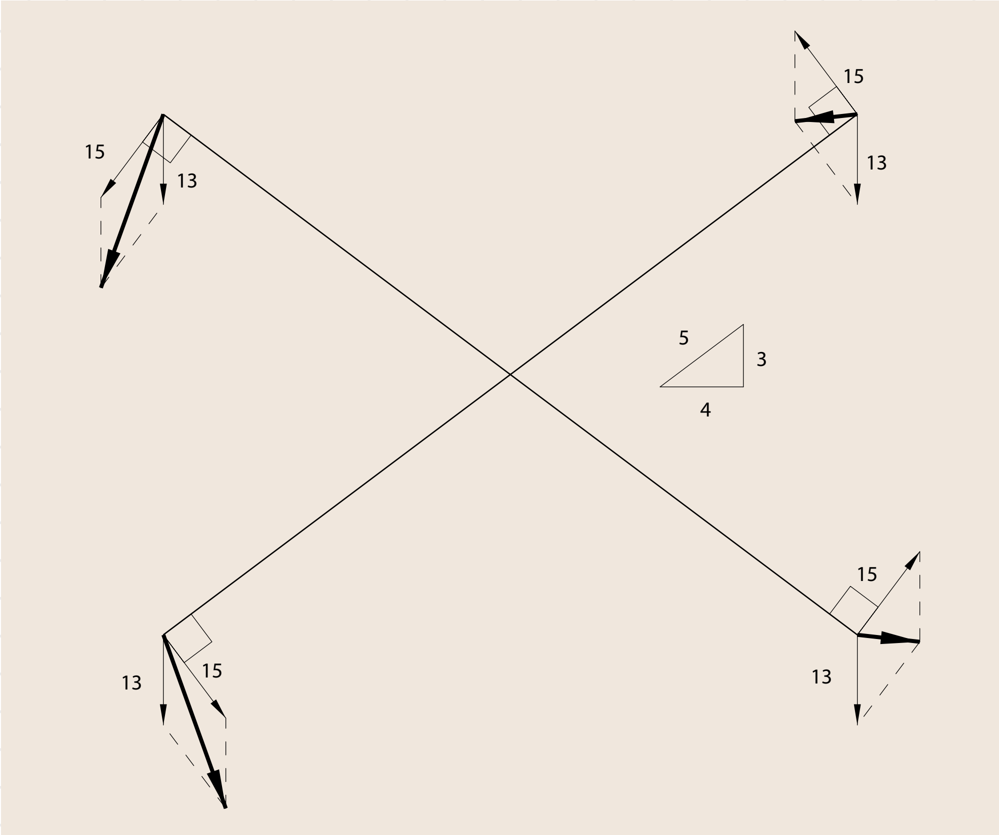
In this approach, the vertical force and torque are shared equally between the four bolts.

Each bolt is subject to:

Vertical force $F_{\mathrm{vert,Ed}} = \dfrac{V_{\mathrm{Ed}}}{4} = \dfrac{52}{4} = 13 \text{ kN}$

Inclined force due to torque $F_{\mathrm{inc,Ed}} = \dfrac{3.75}{4 \times 62.5} \times 10^3 = 15 \text{ kN}$

The angle of inclination of the force will vary, as shown below.



The forces shown are in kN.

The largest resultant force is on the lower bolt on the LH side, which may be calculated as:

Vertical component of resultant $= 13 + \frac{4}{5} \times 15 = 25$ kN

Horizontal component of resultant $= \frac{3}{5} \times 15 = 9$ kN

Hence, the resultant shear force on the bolt $F_{v,Ed} = \sqrt{25^2 + 9^2} = 26.6$ kN

It can be demonstrated in the normal way that this is less than the design resistance of a M20 class 8.8 bolt in a 10 mm endplate.

6.4 Approach two

In this approach the combination of vertical force (52 kN) and torque (3.75 kNm) is replaced by an equivalent vertical force of 52 kN acting at an eccentricity of 0.0721 m (3.75 kNm / 52 kN) from the centre of the bolt group.

The resistance of the connection to this eccentric force can be calculated as outlined in SCI publication P358 for fin plate connections with two vertical lines of bolts.

For M20 class 8.8 bolts in a 10 mm endplate, the lowest resistance given by the relevant design checks given in P358 is 184 kN and thus the connection is adequate for the 52 kN force at this eccentricity.

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DESIGN OF STEEL BEAMS IN TORSION

In most steel-framed structures, beams are subject only to bending and not to torsion. Designers are therefore much less familiar with evaluating torsional effects when they do occur and with determining the resistance to torsion, particularly in conjunction with bending. This guide explains the basic behaviour of beams in torsion and provides formulae and graphs for evaluating the effects of torsion. Practical guidance is given on the design for torsional resistance, in accordance with Eurocode 3, including the interaction of torsion with bending resistance and buckling resistance.

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